Pricing Voluntary Disclosure

Shuting Hou, Rui Sun, and Hulai Zhang*

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Abstract

Many firms make non-regulatory information disclosure, such as ESG reporting, to the market. We develop and estimate a dynamic stochastic general equilibrium model to analyze firms' voluntary disclosure behaviors. Our estimation suggests that firms refrain from voluntary disclosure when households perceive a high level of disclosure uncertainty. Additionally, it indicates that households ought to reduce their consumption and stock market investments as the uncertainty in disclosure increases. Our findings show an increase in both the risk premium and volatility of stocks with rising disclosure uncertainty.

JEL Classification: G12, G18, D72, P16

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*Shuting Hou, School of Business, East China University of Science and Technology. Rui Sun, Haas School of Business, University of California, Berkeley. Hulai Zhang, Department of Finance, Tilburg University and ESCP Business School. Our correspondences are shutinghou@ecust.edu.cn, ruisun233@berkeley.edu, and h.zhang_4@tilburguniversity.edu.

1 Introduction

The impact of uncertainty about voluntary disclosure on decision-making processes in firms and households is still an important and unresolved problem in literature. Voluntary disclosure should have minimal impact on firms' and households' decisions in a perfect market (Aghamolla and Smith, 2023). This assumption is particularly relevant in financial markets, as both parties frequently prioritize urgent operational and investment decisions over the speculative aspects of voluntary disclosures. Research suggests that firms and households weigh the risks of voluntary disclosure when planning their future undertakings (Kim et al., 2018; Kumar et al., 2017; Pawliczek et al., 2022). Consequently, research is needed to examine the impact of voluntary disclosure on household consumption and investment decisions, thereby influencing stock market dynamics.

Based on the model set out by Pástor and Veronesi (2013), our study looks at the effects of voluntary disclosure in a dynamic setting with firms and households interacting with each other. We add an additional firm behavior to the model: voluntary information disclosure. Firms can choose to disclose their own information voluntarily to the market in order to correct market beliefs. Our model features the learning behaviors of both firms and households. Households learn about firm's performance, while firms learn about households' belief on them. Voluntary disclosure affects households' consumption and investment, thus affects stock returns of the firm.

Why do firms voluntarily disclose their private information to the market? Graham et al. (2005) demonstrates that managers often rely on voluntary disclosures to reduce information risk. This strategy reduces the information risk premium or increase stock prices by addressing the information asymmetry between managers and investors. Dhaliwal et al. (2011) shows that voluntary disclosures, particularly in Corporate Social Responsibility (CSR), enable firms to establish stronger relationships with institutional investors, thereby reducing costs of capital. The information disclosure is particularly common among companies that are currently experiencing financial constraints. Shroff et al. (2013) finds the same relationship in the pre-offering disclosures. Voluntary disclosures enhance the predictability of firm prospects, which makes the firm more resilient of negative news. Skinner (1994) posits that benefits of disclosing negative news, despite its immediate adverse connotations, can be outweighed by the downside of lowered expectations on cash flows. Managerial voluntary disclosures increase transparency between investors and the firm, thus increase stock prices. These findings point out the importance of voluntary disclosure in influencing stock market. Our paper investigates how voluntary information disclosure affect stock market with a learning model.

The primary mechanism in our model, by which the voluntary disclosure affects household behavior, is the expected outcomes of the disclosure. This disclosure generates the risk of unforeseen outcomes, and subsequent market volatility, thereby changing household beliefs. Consequently, households adjust their optimal consumption levels and asset allocation based on their updated beliefs. This finding is consistent with the asset pricing literature on mandatory information disclosure. Research by Asness et al. (2013) and Borovicka (2020), in a long-term risk model, illustrate that households integrate the uncertainty of future corporate disclosure into their current consumption decisions. This form of uncertainty generates a notable risk premium, and impacts household consumption and investment.

Most existing research on the uncertainty of voluntary disclosure suggests that heightened disclosure uncertainty compels households to reduce their current consumption and venture capital investments, consequently elevating their savings rate. This observation is corroborated by studies such as Bansal and Yaron (2004) and Attanasio and Weber (2010). Theoretical models, including the real option effect and the precautionary saving effect, offer further insight into this phenomenon. The real option effect, for instance, posits that risk-averse families, wary of potential future income or investment environment deterioration, are inclined to increase their cash holdings in response to rising risks associated with voluntary disclosure. Subsequently, they modify their future consumption levels and investment decisions in line with the evolving disclosure landscape, as elucidated in research by Ai and Kiku (2013), Bernanke (1983), Bernanke and Gertler (1990), and Bernanke et al. (1999). Conversely, the precautionary saving effect, grounded in Keynes' macroeconomic theory, asserts that households are inclined to increase their savings as a hedge against uncertain future events, such as those induced by voluntary disclosure uncertainty. This tendency is supported by findings from Van Der Ploeg (1993) and Choi et al. (2017). As uncertainty intensifies, the proportion allocated to precautionary savings surges, while consumption and investment proportionately diminish. Notably, in certain instances, voluntary disclosure uncertainty might also trigger an increase in consumption, particularly in scenarios where the disclosure is related to business expansion, prompting households concerned about potential equity dilution to elevate their current expenditure on non-durable goods.

Moreover, within our framework, stock prices may be influenced directly or indirectly by the uncertainty of voluntary disclosure. The indirect influence occurs as disclosure uncertainty determines stock price changes through the consumption and investment choices of households. While the Capital Asset Pricing Model (CAPM) is commonly utilized to analyze the interplay among consumption, investment, and stock price, the incorporation of disclosure uncertainty within this framework remains relatively underexplored (Fama and French, 2015). Researchers like Segal et al. (2015) and Dew-Becker et al. (2017) categorize macroeconomic uncertainty into 'good' and 'bad' types, employing a recursive utility function to examine the effects of uncertainty on consumption, investment, and stock price. This paper offers a novel approach to understanding the relationship between disclosure uncertainty, consumption, and stock price, emphasizing the role of information friction. Additionally, much of the existing research concentrates on the nexus between disclosure uncertainty and either consumption or investment in isolation, typically within a partial equilibrium context.

While these studies provide substantial empirical evidence on the correlation between disclosure uncertainty and stock returns, there is a noticeable lack of theoretical underpinning.

In exploring the relationship between disclosure uncertainty and investment, parallels can be drawn to consumption choices. Classical asset pricing models typically predict that disclosure uncertainty diminishes investment. The theoretical discourse in this area primarily revolves around the risk premium effect and the real option effect. The risk premium effect posits that disclosure uncertainty escalates the default probability of corporate loans and amplifies the default risk, particularly in the lower tail of the risk distribution. This leads to banks increasing loan interest rates, thereby elevating enterprises' debt financing costs and potentially causing a further downgrade in their credit ratings. This cascading effect, a fundamental concept in credit default theory and the design of credit derivatives, perpetuates through the leverage effect. Considering uncertainty in a broader context, Bloom (2009) have demonstrated that increased uncertainty correlates with reduced investment levels and employment numbers. In a similar vein, empirical research by Baker et al. (2016) confirms the negative association between corporate disclosure uncertainty and investment. Additionally, Gulen and Ion (2016) have investigated the sensitivity of corporate cash holdings in response to varying degrees of corporate disclosure uncertainty.

Conversely, some scholars have identified scenarios where an increase in voluntary disclosure uncertainty may actually stimulate investment. Lund (2005) illustrates that the growth option effect enables R&D-focused enterprises to leverage potential opportunities presented by voluntary disclosure, leading to increased R&D expenditure. Although R&D initiatives carry the risk of failure, with only the sunk cost of R&D being lost, successful R&D outcomes can generate substantial wealth for the enterprises. Furthermore, Abel (1983), Guiso and Parigi (1999), and Bloom et al. (2007) suggest that, under certain regularity conditions, firms possess adequate flexibility in determining investment scale and product pricing. This flexibility allows firms to identify optimal investment strategies that foster average output growth in various uncertain environments. However, the applicability of these findings is moderated by firms' adjustment costs. Therefore, in the context of voluntary disclosure, while the impact may not be immediately evident in the short term, it becomes more pronounced over medium and long-term horizons.

Voluntary disclosure uncertainty can exert a direct influence on both the returns and the stochastic discount factor of households. In our model, voluntary disclosure has the potential to alter the external environment of firms, thereby directly impacting firm operations through stock returns. Additionally, the negative effect of voluntary disclosure uncertainty on the market is non-diversifiable, compelling households to seek compensation for this risk by correspondingly increasing the discount rate. As the risk associated with voluntary disclosure escalates, there is an ensuing rise in the volatility of the firm's future yield level and the stochastic discount factor. Analogous to the dividend discount model, the stock price is contingent upon its future cash flows and the stochastic discount factor. Fluctuations in these two factors invariably lead to an increase in the stock risk premium and price volatility. The model developed by Pástor and Veronesi (2013) delineates the stochastic process of disclosure impacts on corporate profit rates and integrates disclosure uncertainty into the stock price via the discount factor, demonstrating that disclosure uncertainty can enhance stock returns and the risk index. Additionally, Manela and Moreira (2017) utilize stochastic tax rates as a proxy for disclosure uncertainty, which can be perceived as external voluntary disclosure. Employing a dynamic general equilibrium model, they analyze its impact on asset pricing. The model, an extension of the Lucas asset pricing framework, incorporates a smooth consumption tax following a two-state Markov chain, establishing that taxation systematically influences stock prices, resulting in increased volatility of expected and real returns. On the empirical front, studies by Bonaime et al. (2018) and Jens (2017) have explored the relationship between voluntary disclosure uncertainty and firm mergers and acquisitions using panel data from the U.S. Their risk factor analysis indicates that disclosure uncertainty functions as a systemic risk factor at the macro level.

This paper makes three significant contributions to the existing literature. First, we develop a comprehensive model that nests both households and firms, incorporating habit formation into the households' decision-making process. This framework enables us to derive a more expansive asset pricing model that encompasses disclosure uncertainty, consumption, and the firm's decision-making process regarding voluntary disclosure. Our approach extends the model by Pástor and Veronesi (2013) by integrating households' consumption, thereby reflecting the aggregate effects of disclosure uncertainty on stock price changes after considering the impact of disclosure uncertainty on the household's portfolio choice. Furthermore, our analysis delves into the effects of disclosure uncertainty on household consumption and asset allocation, with comparative statics on stock prices that aptly capture the distinctive attributes of the U.S. stock market, thus significantly enriching the theoretical insights of existing models. Second, we focus on the utility of investors at the end of the disclosure period, centering on the firm's optimal disclosure choices aimed at maximizing investors' utility post-voluntary disclosure. This aspect provides a novel perspective on the strategic decision-making of firms in the context of household welfare. Third, our estimation results align closely with empirical evidence observed in the U.S. For instance, the growth rate derived from the disclosure threshold mirrors the actual growth rate in the U.S., and the observed relationship between the proportion of venture capital and stock price resonates with the prominent presence of retail investors in the U.S. stock market.

The paper is organized as follows. Section 2 presents the model and examines the implications of the model. Section 3 describes the data in this paper and our calibration. Section 4 presents the quantitative findings. Section 5 assesses the robustness of the model. Section 6 concludes.

2 Model

This section describes our dynamic model for analyzing voluntary disclosure. The model has two parties: a representative household and a representative firm. The household allocates its wealth between the risky firm and a safe asset. Their objective is to maximize the utility derived from consumption, taking into account a slow-moving habit. The firm employs household equity finance for its daily operations. The time spans from $[0, T]$. At an exogenous time $t^* \in [0, T]$, the firm decides whether or not to make a voluntary disclosure reflecting its performance. Suppose that the household owns the firm. Hence, the firm's objective aligns with the household's, considering the expenses associated with voluntary disclosure.

2.1 Firms

A continuum of representative firms are indexed by $j \in [0, 1]$. The firm relies entirely on equity financing from households for its operations. The firm is fully owned by the household. The firm's book asset, S_t^j, comes from the household's allocation of its total asset A_t. Thus, $S_t^j = (1 - s_t)A_t$, where $1 - s_t$ is the share of household's asset invested in the firm. The process of firm's book asset is

$$dS_t^j = S_t^j dr_t^j = (1 - s_t)A_t dr_t^j,$$

where dr_t^j is the reported return of firm j, and it follows the process

$$dr_t^j = (\mu + a_t)dt + (\sigma_r + \iota_t)dW_t + \sigma_e dW_t^j. \tag{1}$$

$\mu + a_t$ represents the drift term, measuring the average return of the firm. Firm's decision to reveal optional information affects the stock return through the term a_t. If there is no disclosure, firm's return sees no additional impact, e.g. $a_t = 0$. The diffusion term is given by $\sigma_r + \iota_t$, where ι_t captures the time-varying volatility of returns and is bounded with mean 0 and variance τ_ι^2. W_t^j is the idiosyncratic volatility of returns for firm j, independent of W_t.

The firm's decision of voluntary disclosure at the exogenous time t^* is based on the comparison between the benefit (captured by a_t) and cost (captured by C) of voluntary disclosure. If the firm conducts voluntary disclosure at t^*, there will be no further disclosure after t^*. The return on asset will be added by a non-zero value $a_t = a_1$ on and after t^*. We normalize the effect of no voluntary disclosure as zero, e.g. $a_t = 0$. Thus, a_1 measures the net impact of voluntary disclosure, relative to no disclosure on the firm's return. a_t is a private information of the firm, which is not observed by households. We assume households learn a_t according to a Bayesian learning process. Their prior of a_t at $t = 0$ is a normal distribution $N(0, \tau_a^2)$. The variance term τ_a^2 reflects the prior knowledge about the fluctuation of the impact by voluntary disclosure on stock returns. Note that both $a_0 = 0$ and a_1 are not revealed to the household. Instead, households only observe an aggregate signal for a_t: $d\chi_t = (\mu + a_t)dt + (\sigma_r + \iota_t)dW_t$.

2.2 Households

A continuum of representative households are index by $i \in [0, 1]$. Their total asset at time t is A_{it}. We omit the subscript i for simplicity in the following discussion. At time t, the household allocates its total asset between the risky firm and a risk-free asset. The proportion allocated to the risk-free asset is s_t, and the proportion allocated to the equity of the firm is $1 - s_t$. The return of the risk-free asset is set to be r_f. The process of household's total asset is

$$dA_t = (1 - s_t)A_t dr_t + s_t r_f A_t dt - c_t dt. \tag{2}$$

where $r_t = \int_0^1 r_t^j dj$ denotes the average reported return from firms. Meanwhile, the household can not observe the impact of voluntary disclosure a_t, but only receive an aggregate signal $d\chi_t$

in each period. We assume the household updates its belief about a_t by Bayesian learning.

By Bayes rule, the households' posterior belief of a_t during the interval $[0, T]$ can be demonstrated as $N(\hat{a}_t, \hat{\tau}^2_{a_t})$, where $\hat{a}_t$ and $\hat{\tau}^2_{a_t}$ stand for the posterior mean and variance respectively. In particular, when $t \leq t^*$, $\hat{a}_t$ and $\hat{\tau}^2_{a_t}$ can be written as

$$\begin{aligned} d\hat{a}_t &= \hat{\tau}^2_{a_t} \frac{dr_t - E_t(dr_t)}{(\sigma_r + \iota_t)(\sigma_r^2 + \tau_\iota^2)^{\frac{1}{2}}} \\ \hat{\tau}^2_{a_t} &= \frac{1}{\frac{1}{\tau_a^2} + \frac{t}{\sigma_r^2 + \tau_\iota^2}} \end{aligned} \tag{3}$$

where $\frac{dr_t - E_t(dr_t)}{\sigma_r + \iota_t}$ is the expectation error. If there is no voluntary disclosure at time t^*, the households' posterior belief of a_t can still be described by Equation (3). However, when a voluntary disclosure happens, the posterior mean will jump to zero at t^*. After t^*, the posterior mean also evolves as Equation (3). However, the new posterior variance conditional on the voluntary disclosure becomes

$$\hat{\tau}^2_{a_t} = \frac{1}{\frac{1}{\tau_a^2} + (t - t^*)\frac{1}{\sigma_r^2 + \tau_\iota^2}}, t > t^* \tag{4}$$

In addition, household's utility is based on their consumptions and with a persistent habit, as in Constantinides (1990). The expected utility of the household is

$$\mathbb{E}_0 \int_0^\infty e^{-\beta t} \frac{(c_t - x_t)^{1-\phi}}{1-\phi} dt$$

where β is the subjective discount rate, ϕ is the risk aversion, c_t is the household's consumption, and x_t is the consumption habit. The habit at time t is defined as

$$x_t = e^{-b_1 t}\bar{x} + b_2 \int_0^t e^{b_1(u-t)} c_u du,$$

where $\bar{x}$ is the initial habit of the household at time 0. b_1 and b_2 are two parameters to control the weights of intinial habit and consumptions of each period in the habit formation at time t. Household's initial asset is strictly positive, e.g. $A_0 > 0$.

Since firms are assumed to be owned by households. Firms will evaluate the expected effects of voluntary disclosure on the household's utility. Only when the expected utility caused by a voluntary disclosure is bigger than no disclosure, will the firm disclose voluntarily at t^*. Moreover, the firm's objective is to maximize the households' assets at the end of the interval $[0, T]$. Thus, the objective function of the firm at t^* is

$$V(A_{t^*}, x_{t^*}) = \int_{t^*}^{T} \frac{(\bar{c}_t - \bar{x}_t)^{1-\phi}}{1-\phi} dt$$

where $\bar{c}_t$ and $\bar{x}_t$ denote the optimal consumption rate and habit formation according to the household's problem, which will be defined below. To capture the cost of voluntary disclosure, we assume that the voluntary disclosure will put downward pressure on the household's utility denoted by a disclosure cost of C. The possible reasons for this cost can be the internal conflicts and information processing costs caused by the voluntary disclosure. In most cases, these factors will pull down the household's utility. Therefore, the voluntary disclosure problem at t^* faced by the firm can be presented in the following way.

$$max\{E_{t^*}[V(A_{t^*}, x_{t^*})|\text{Old}], E_{t^*}[CV(A_{t^*}, x_{t^*})|\text{New}]\}$$

where the disclosure cost C is set to follow a log-normal distribution independent of the Brownian motions defined in the process of stock return. τ_c denotes the fluctuation of the cost of the voluntary disclosure(Pástor and Veronesi, 2013). Note that τ_c is the second source of disclosure uncertainty in addition to τ_a. $\ln(C)$ follows a normal distribution[1].

$$\ln(C) \sim N(-\frac{1}{2}\tau_c^2, \tau_c^2)$$

[1]This distribution setting can ensure that the expected cost of voluntary disclosure is $E(C) = 1$.

Before presenting the solution to the firm's disclosure choice, we first characterize of the firm's book value at the end of the disclosure period in the following lemma.

Lemma 1. *Suppose the firm decides whether to conduct voluntary disclosue at time $t^* \in [0, T]$. Let the total book asset of all firms at t be $S_t = \int_0^1 S_t^j dj$. Then,*

$$S_T = S_{t^*} \times \exp\left\{(1-\bar{s})\left[(\mu + a - \frac{(1-\bar{s})\sigma_r^2}{2})(T - t^*) + \sigma_r(W_T - W_{t^*}) + \int_{t^*}^{T} \iota_u dW_u\right]\right\}$$

where $\bar{s}$ is the household's optimal savings. $a = a_1$ if the voluntary disclosure occurs, and $a = 0$ if no voluntary disclosure.

2.3 Model Solution

Before calibrating the model using real data, we need to establish the relationship between the householder's consumption and portfolio choices by solving the optimal dynamic system describing the model.

Firstly, by solving the household problem subject to the evolution of assets defined by Equation (2), we can have an analytic solution to the household's optimal asset holdings at the end of the $[0, T]$. We summarize the key results in the following Proposition

Proposition 1. *The household's optimal asset holdings after the disclosure date t^* can be expressed in the following equation.*

$$A_s = \frac{x_s}{r_f + b_1 - b_2} + (A_{t^*} - \frac{x_{t^*}}{r_f + b_1 - b_2})e^{[(Z_2 - \frac{Z_1^2(\sigma_r^2 + \tau_\iota^2)}{2})(s - t^*) + Z_1\sigma_r(W_s - W_{t^*}) + Z_1\int_{t^*}^{s} \iota_u dW_u]}, s \geq t^* \tag{5}$$

where the parameters Z_1 and Z_2 are defined as

$$Z_1 = \frac{\mu_r + a_{t^*} - r_f}{\phi(\sigma_r^2 + \tau_\iota^2)}, Z_2 = \frac{r_f - \beta}{\phi} + \frac{(\mu_r + a_{t^*} - r_f)^2(1 + \phi)}{2\phi^2(\sigma_r^2 + \tau_\iota^2)}$$

where the parameters are outlined in the model description, and a_{t^*} is the impact of the manager's disclosure choice on stock return at t^*.

Combing the household's optimal asset holdings (e.g. Equation (5)), habit formation process, and the household's expected utility function, we can have the value function of a household when A_t follows the optimal process given by Equation (5). Then the household's value function $V(A_t, x_t)$ is equal to

$$V(A_t, x_t) = E_t \int_t^{\infty} e^{-\beta(u-t)} \frac{(\bar{c}_s - \bar{x}_s)^{1-\phi}}{1-\phi} du = \frac{(r_f + b_1 - b_2) Z_3^{-\phi}}{(r_f + b_1)(1-\phi)} (A_t - \frac{x_t}{r_f + b_1 - b_2})^{1-\phi}$$

where Z_3 is defined as

$$Z_3 = [\frac{r_f + b_1 - b_2}{(r_f + b_1)\phi}][\beta - (1-\phi) r_f - \frac{(1-\phi)(\mu_r + a_{t^*} - r_f)^2}{2\phi(\sigma_r^2 + \tau_\iota^2)}]$$

And $\bar{c}_t$ and $\bar{s}_t$ are the optimal consumption and portfolio holding of the representative household at t, both of which are solved from the following dynamic programming problem.

$$(\bar{c}_t, \bar{s}_t) = \underset{(c_t, s_t)}{\operatorname{argmax}} H_t = \int_0^{\Delta t} e^{-\beta u} \frac{(c_u - x_u)^{1-\phi}}{1-\phi} du + e^{-\beta \Delta t} U_t(A_{t+\Delta t}, x_{t+\Delta t})$$

where Δt denotes a short time. Using Ito's lemma, differentiating the value function with respect to Δt gives us the following condition.

$$\frac{dH_t}{d\Delta t} = Z_{4t} + e^{-\beta \Delta t}(1 - s_t) A_t U_{tA} (\sigma_r + \iota_t) \frac{dW_t}{d\Delta t}$$

$$\begin{aligned} Z_{4t} = e^{-\beta t}(&\frac{(c_t - x_t)^{1-\phi}}{1-\phi} - \beta U_t + U_{tA}(((1-s_t)(\mu + a_{t^*} - r_f) + r_f) A_t - c_t) \\ &+ U_{tAA} A_t^2 (1-s_t)^2 \frac{(\sigma_r^2 + \tau_\iota^2)}{2} + (b_1 c_t - b_2 x_t) U_{tx}) \end{aligned}$$

where U_{tA} and U_{tx} represent the first order derivative of value function U_t with respect to

A_t and x_t. U_{tAA} stands for the second order derivative of U_t with respect to A_t.

Since W_t is a Brownian motion, taking the expectation of the derivative of the value function with respect to time can get rid of the diffusion term. We only need to focus on the drift term given by Z_{4t}. The first-order condition of Z_{4t} with respect to c_t and s_t can give us the optimal consumption and saving rate. We summarized the household's optimal choices in the following Proposition.

Proposition 2. *The optimal consumption $\bar{c}_t$ and saving rate $\bar{s}_t$ of the household are given as*

$$\bar{c}_t = x_t + Z_3(A_t - \frac{x_t}{r_f + b_1 - b_2}), \bar{s}_t = 1 - Z_1[1 - \frac{x_t}{A_t(r_f + b_1 - b_2)}]$$

Combing the analytic solution to asset holdings of a household in Equation (5), we can obtain the law of motion of her optimal consumption rate by differentiating the optimal consumption with respect to time.

$$\frac{d\bar{c}_t}{\bar{c}_t} = [Z_2 + b_2 - \frac{(Z_2 + b_1)x_t}{\bar{c}_t}]dt + (1 - \frac{x_t}{\bar{c}_t})Z_1(\sigma_r + \iota_t)dW_t \tag{6}$$

Define $h_t = \frac{x_t}{c_t}$ as the ratio of habit to consumption. Combing the optimal solution to consumption and saving rate, we can derive the stationary level of this ratio denoted as $\bar{h}$

$$\bar{h} = \frac{Z_2 + b_1 - [(Z_2 + b_1)^2 - 4Z_1^2(\sigma_r + \tau_\iota^2)b_2]^{\frac{1}{2}}}{2Z_1^2(\sigma_r^2 + \tau_\iota^2)} \tag{7}$$

And $\bar{s}_t$ is a constant when h_t reaches the stationary level $\bar{h}$.

The threshold of a_1 for voluntary disclosure and the corresponding probability are the two primary variables in determining asset prices. Both variables can affect the jump of stock prices at the disclosure date t^* and have a long-term impact on stock prices afterward.

In the equilibrium, $h_t = \frac{x_t}{c_t}$ will converge to a constant $\bar{h}$ given by Equation (7). Thus, we can deduce that the habit to total assets ratio $\frac{x_t}{A_t}$ also converges to a constant given by

Proposition 2. With a constant $\frac{x_t}{A_t}$, the optimal asset holdings of households during $[0, T]$ in Equation (5) can be transformed into

$$A_s = A_{t^*} e^{[(Z_2 - \frac{Z_1^2(\sigma_r^2 + \tau_\iota^2)}{2})(s-t^*) + Z_1\sigma_r(W_s - W_{t^*}) + Z_1 \int_{t^*}^{s} \iota_u dW_u]}, s \in [t^*, T]$$

At t^*, if a voluntary disclosure occurs, the households will reset their belief, and the disclosure variable at t^* will still follow the prior distribution $N(0, \tau_a^2)$. However, if there is no disclosure, a_{t^*} will satisfy the Bayesian posterior distribution $N(\hat{a}_{t^*}, \hat{\tau}_{a_{t^*}}^2)$. Moreover, by plugging Equation (7) into the voluntary disclosure problem defined in Section 3.2, we can obtain the following Proposition.

Proposition 3. *Voluntary disclosure will be implemented only when $\hat{a}_{t^*} < \underline{a}(C)$, where $\underline{a}(C)$ can be numerically solved by equalizing $E_{t^*}[V(A_{t^*}, x_{t^*})|Old]$ to $E_{t^*}[CV(A_{t^*}, x_{t^*})|New]$.*

Given the threshold $\underline{a}(C)$, the distribution of cost variable C, and the household's posterior belief of the disclosure variable a_t, we can further derive the probability that a voluntary disclosure occurs at time t^* defined by $q(\hat{a}_{t^*})$.

$$q(\hat{a}_{t^*}) = Prob(\hat{a}_{t^*} < \underline{a}(C)) = F_N(-\frac{\tau_c^2}{2}, \tau_c^2; \underline{a}^{-1}(C)) \tag{8}$$

where $\underline{a}^{-1}(C)$ is the inverse function of $\underline{a}(C)$, and $F_N(x, y; z)$ denotes the cumulative distribution of a normal distribution with mean x and variance y at the point z.

Proposition 3 describes how the firm's strategy about voluntary disclosure, $\hat{a}_{t^*}$ captures the effect of the voluntary disclosure at t^* on stock return. We can regard $\underline{a}(C)$ as the return target of the manager. Therefore, the manager will conduct voluntary disclosure when she expects no disclosure can not achieve the target. However, since the disclosure cost C is stochastic, meaning it can not be measured accurately in advance, the decision about voluntary disclosure can also be stochastic. All these conditions make the voluntary disclosure uncertain.

According to the classical asset pricing theory, state price density, stock return, stock return volatility, and risk-free interest rate are the most important components to describe a stock market. This paper uses real data to calibrate the risk-free interest rate. The state price density and stock return can be obtained by state pricing and the Brownian motions in the stock return process. However, the volatility of stock prices might not be a good proxy for the risk. Some recent studies demonstrate that the jump risk of the stock price is also an essential part of the stock market risk. In particular, Johannes (2004) shows that the big jump in stock prices can be associated with unexpected macro fluctuations. Next, we move to define the state price density, stock return, volatility of stock prices, and the jump risk.

Based on the Euler equation, the market value of stock j at time t during the $[0, T]$ can be defined as

$$Q_t^j = E_t(\frac{P_T}{P_t} S_T^j)$$

where P_T and P_t denote the state price density at T and t, and Q_t^j is the market value of the stock issued by firm j. Since firms are identical and there is a continuum of firms with measure unity, we remove the index j in the following analysis. Solving the model gives us the state price density of stocks P_t as stated in the following proposition.

Proposition 4. *Before the disclosure date t^*, the state price density of firm stocks can be presented by*

$$P_t = S_t^{-\phi}(Prob_t^{New} E_t^{New} + (1 - Prob_t^{Old}) E_t^{Old})$$

where $Prob_t^{Yes}$, $Prob_t^{No}$, E_t^{Yes}, and E_t^{No} are defined in the following equations[2]

$$
\begin{aligned}
Prob_t^{Yes} &= F_N(\hat{a}_t - \phi\hat{\tau}_t^2(t^* - t) + \frac{{\tau_c}^2}{2(1-\phi)(T-t^*)}, {\hat{\tau}_t}^2 - \hat{\tau}_{t^*}^2 + \frac{{\tau_c}^2}{(1-\phi)^2(T-t^*)^2}); a(\underline{0})) \\
Prob_t^{No} &= F_N(\hat{a}_t - \phi(\hat{\tau}_t^2(T-t) - \hat{\tau}_{t^*}^2(T-t^*)) + \frac{{\tau_c}^2}{2(1-\phi)(T-t^*)}, {\hat{\tau}_t}^2 - {\hat{\tau_{t^*}}}^2 + \frac{{\tau_c}^2}{(1-\phi)^2(T-t^*)^2}); a(\underline{0})) \\
E_t^{Yes} &= e^{-\phi(1-s_t)(\mu-(1+\phi)(1-\bar{s})(\sigma_r^2+\tau_\iota^2))(T-t^*)+\frac{\phi^2(1-\bar{s})^2\tau_a^2(T-t^*)^2}{2}-\phi E_t(ln(S_{t^*}))+\frac{\phi^2}{2}Var_t(ln(S_{t^*}))} \\
E_t^{No} &= e^{-\phi(1-\bar{s})(\mu-(1+\phi)(1-\bar{s})(\sigma_r^2+\tau_\iota^2))(T-t^*)+\frac{\phi^2(1-\bar{s})^2\hat{\tau}_{a_{t^*}}^2(T-t^*)^2}{2}-\phi E_t(\Psi_{t^*})+\frac{\phi^2}{2}Var_t(\Psi_{t^*}^2)}
\end{aligned}
$$

where Ψ_t is defined as $\Psi_t = (1-\bar{s})\hat{a}_t(T-t) + ln(S_t)$, and the expressions of expectation and variance conditional on information set in period t^* above are given by

$$
\begin{aligned}
E_t(ln(S_{t^*})) &= ln(S_t) + (1-\bar{s})(\mu + \hat{a}_t - \frac{1}{2}(\hat{a}_t^2 + \tau_\iota^2))(t^* - t) \\
Var_t(ln(S_{t^*})) &= (1-\bar{s})^2((t^*-t)^2\hat{\tau}_t^2 + (\sigma_r^2 + \tau_\iota^2)(t^* - t)) \\
Var_t(\Psi_{t^*}) &= (1-\bar{s})^2(\hat{\tau}_{a_t}^2(T-t)^2 - (T-t^*)^2\hat{\tau}_{t^*}^2 + (\sigma_r^2 + \tau_\iota^2)(t^* - t))
\end{aligned}
$$

Different from Pástor and Veronesi (2009), the mean and diffusion term of the growth rate of the aggregate book value of stocks S_t varies with households' portfolio choice rather than being a constant because the household can optimize her portfolio choice between risk-free saving and stock holdings. Moreover, this modification will also change the parameters that determine the state price density and the market value of stocks. To simplify our analysis, we assume that households make decisions before the realization of the book value of stocks. In other words, the optimal fraction of stock holdings in total assets of the household is stochastic in her problems. In the first case, we derive the evolution process of the state price density by Ito's lemma.

$$\frac{dP_t}{P_t} = -\sigma_{P,t}d\hat{W}_t + K_P 1_{t=t^*} \tag{9}$$

[2] In the purpose of brevity, the detailed derivations in this section are included in the Appendix.

where $\hat{W}_t = \frac{dr_t - E_t(dr_t)}{\sigma_r + \iota_t}$ can be regarded as an "expected error", and $1_{t=t^*}$ is an indicator function equaling to 1 when $t = t^*$ and 0 otherwise. $\sigma_{P,t}$ is the volatility of state price density, which can be defined as

$$\sigma_{P,t} = \begin{cases} \phi(1-s_t)(\sigma_r+\iota_t) - \frac{R_{\hat{a}_t}(\hat{a}_t)}{R((\hat{a}_t)}\hat{\tau}_{at}^2(1-s_t)^{-1}(\sigma_r+\iota_t)^{-1}, t \leq t^* \\ \phi(1-s_t)(\sigma_r+\iota_t) + (T-t)\hat{\tau}_{at}^2(1-s_t)^{-1}(\sigma_r+\iota_t)^{-1}, t \geq t^* \end{cases}$$

where $R(\hat{a}_t) = Prob_t^{Change}E_t^{Change} + (1 - Prob_t^{Stay})E_t^{Stay}$, $R_{\hat{a}_t}(\hat{a}_t)$ is the derivative of $R(\hat{a}_t)$ with respect to $\hat{a}_t$, K_P stands for the jumps in the state price density, which follows the expressions below.

$$K_P = \begin{cases} \frac{(1-H(\hat{a}_{t^*}))(1-q(\hat{a}_{t^*}))}{q(\hat{a}_{t^*})+(1-q(\hat{a}_{t^*}))H(\hat{a}_{t^*})}, & \text{Disclosure happens} \\ \frac{(H(\hat{a}_{t^*}))-1)q(\hat{a}_{t^*})}{(1-q(\hat{a}_{t^*}))H(\hat{a}_{t^*})+q(\hat{a}_{t^*})}, & \text{No disclosure} \end{cases}$$

where the function $H(\hat{a}_{t^*})$ is defined as

$$H(\hat{a}_{t^*}) = e^{-\phi\hat{a}_{t^*}(T-t^*) - \frac{1}{2}\phi^2(T-t^*)^2(\tau_a^2 - \hat{\tau}_{t^*}^2)}$$

From the expression of the jump K_P with and without voluntary disclosure, we can see the uncertainty of voluntary disclosure has a significant impact on the jump in state price density. By a similar derivation, we can calculate the expression of the market value of stocks issued by firm j, and the main results are summarized in the following Proposition.

Proposition 5. *Before t^*, the market value of stocks issued by firm j is defined as*

$$Q_t^j = \frac{R(\hat{a}_t)}{H_2(\hat{a}_t)}S_t^j$$

where the expression $R(\hat{a}_t)$ and $H_2(\hat{a}_t)$ are defined as

$$R(\hat{a}_t) = Prob_t^{Change} E_t^{Change} + (1 - Prob_t^{Stay}) E_t^{Stay}$$
$$H_2(\hat{a}_{t^*}) = Prob_{1t}^{Change} E_{1t}^{Change} + (1 - Prob_{1t}^{Stay}) E_{1t}^{Stay}$$

where $Prob_t^{Change}$, $Prob_t^{Stay}$, E_t^{Change}, and E_t^{Stay} are defined in Proposition 4 , and $Prob_{1t}^{Change}$, $Prob_{1t}^{Stay}$, E_{1t}^{Change}, and E_{1t}^{Stay} are given in following expressions

$$Prob_{1t}^{Change} = F_N(\hat{a}_t + (1-\phi)\hat{\tau}_t{}^2(t^* - t) + \frac{\tau_c{}^2}{2(1-\phi)(T-t^*)}, \hat{\tau}_t^2 - \hat{\tau}_{t^*}^2 + \frac{\tau_c{}^2}{(1-\phi)^2(T-t^*)^2}); a(\underline{0}))$$
$$Prob_{1t}^{Stay} = F_N(\hat{a}_t + (1-\phi)(\hat{\tau}_t{}^2(t^* - t) + \hat{\tau}_t^2(T-t)) + \frac{\tau_c{}^2}{2(1-\phi)(T-t^*)}, \hat{\tau}_t^2 - \hat{\tau}_{t^*}^2 + \frac{\tau_c{}^2}{(1-\phi)^2(T-t^*)^2}); a(\underline{0}))$$
$$E_{1t}^{Change} = e^{(1-\phi)(1-s_t)\mu(T-t)+(1-\phi)\hat{a}_t(t^*-t)+\frac{1}{2}(1-\phi)^2((T-t^*)^2\tau_a^2)+(t^*-t)^2\hat{\tau}_{a_t}^2)-\frac{1}{2}\phi(1-\phi)(1-s_t)^2(\sigma_r^2+\tau_\iota^2)(T-t)}$$
$$E_t^{stay} = e^{(1-\phi)(1-s_t)\mu(T-t)+(1-\phi)\hat{a}_t(T-t)+\frac{1}{2}(1-\phi)^2(T-t)^2\hat{\tau}_{a_t}^2-\frac{1}{2}\phi(1-\phi)(1-s_t)^2(\sigma_r^2+\tau_\iota^2)(T-t)}$$

Deriving equation Q_t^j given by Proposition 5 with respect to time can give us the evolution of the market value of the stock issued by firm j.

$$\frac{dQ_t^j}{Q_t^j} = \mu_{Qt}dt + \sigma_{Qt}d\hat{W}_t + K_Q 1_{t=t^*} \tag{10}$$

where $d\hat{W}_t$ is the expected error term defined before, and K_Q is the jump in the evolution of the stock value, which is defined as

$$K_Q = \begin{cases} N(\hat{a}_{t^*}), & \text{Disclosure happens} \\ N(\hat{a}_{t^*})N_1(\hat{a}_{t^*}) + N_1(\hat{a}_{t^*}) - 1, & \text{No disclosure} \end{cases}$$

where $N(\hat{a}_t^*)$ and $N_1(\hat{a}_t^*)$ are defined in the following way[3]

$$N(\hat{a}_{t^*}) = \frac{(1 - q(\hat{a}_{t^*})H(\hat{a}_{t^*})(1 - N_1(\hat{a}_{t^*}))}{q(\hat{a}_{t^*} + (1 - q(\hat{a}_{t^*})H(\hat{a}_{t^*})N_1(\hat{a}_{t^*}))}$$

[3] The detailed derivation of these expressions are in the appendix.

$$N_1(\hat{a}_{t^*}) = e^{(1-\bar{s})\hat{a}_{t^*}(T-t^*)-\frac{1}{2}(1-2\phi)(1-\bar{s})^2(T-t^*)^2(\tau_a^2-\hat{\tau}_{a_{t^*}}^2)}$$

Furthermore, the expressions of the volatility and expectation of the stock value are given as

$$\mu_{Q,t} = \begin{cases} \sigma_{P,t}\sigma_{Q,t}, t \leq t^* \\ \phi((1-s_t)(\sigma_r+\tau_\iota)+(T-t)\hat{\tau}_{a_t}^2(1-s_t)(\sigma_r+\tau_\iota))^{-2}, t \geq t^* \end{cases}$$

$$\sigma_{Q,t} = \begin{cases} (1-s_t)(\sigma_r+\iota_t)+(\frac{H_{1\hat{a}_{t^*}}(\hat{a}_{t^*})}{H_1(\hat{a}_{t^*})}-\frac{H_{\hat{a}_{t^*}}(\hat{a}_{t^*})}{H(\hat{a}_{t^*})})\hat{\tau}_{a_t}^2(1-s_t)^{-1}(\sigma_r+\iota_t)^{-1} & \text{if } t < t^* \\ (1-s_t)(\sigma_r+\iota_t)+(T-t)\hat{\tau}_{a_t}^2(1-s_t)^{-1}(\sigma_r+\iota_t)^{-1} & \text{if } t \geq t^* \end{cases}$$

where $H_{\hat{a}_{t^*}}(\hat{a}_{t^*})$ and $H_{1\hat{a}_{t^*}}(\hat{a}_{t^*})$ are corresponding first order derivatives of $H(\hat{a}_{t^*})$ and $H_1(\hat{a}_{t^*})$. According to the expressions above, we find that disclosure uncertainty can affect the expectation, volatility, and jumps in the stock value.

3 Data and Calibration

In this section, we will give a brief introduction to the environment of voluntary disclosure in the U.S., the data used for calibration, and present suggestive empirical evidence. Then, we move to calibrate the model described above and check whether our model can match the evidence or not.

The uncertainty of voluntary disclosure is significantly correlated with economic uncertainty, which has experienced notable fluctuations in the U.S. for various reasons. First, given its dynamic economic development and transformation, the U.S. undergoes periodic phases of economic and financial system reform, such as reforms following the financial crisis of 2008, Dodd-Frank Wall Street Reform, the reform of environmental requirements for manufacturers, the reform of social insurance system, and so on. All these events make the economic uncertainty of the U.S. reach an unprecedented high point, driving up the uncertainty of voluntary disclosure in turn. Moreover, firms' decisions in the U.S. can often be influenced by disclosure changes. Transitions in political leadership and differing priorities between administrations introduce variability. New administrations might introduce distinct economic strategies or adjust prior regulatory frameworks, leading to amplified concerns about voluntary disclosure at the corporate level. Lastly, international events and relations can further sway U.S. policies. Global crises, trade relationships, and geopolitical tensions can lead to shifts in U.S. economic policies, further driving up disclosure uncertainty.

Firms often conduct voluntary disclosure in response to different events: for example, the global financial crisis, trade war, and fluctuations in the real estate market. The firms' reactions to these shocks could significantly influence the economy and financial system for a long time. In particular, the prosperity of Fama-French Market Index in the U.S. from the middle of 2003 to the end of 2007 resulted from a huge increase in the quality of the disclosure of listed firms and investor confidence in the information transmitted by the firms after Sarbanes-Oxley Act. The extreme volatility of stock prices since June 2002 also reflects the overreaction of market participants to firms' disclosure during that time. The existing literature adopts mainly an event study framework to analyze stock price changes before and after disclosures. However, few scholars have paid attention to adjusting the potential uncertainty when the firm's information can not be perfectly transmitted to investors by disclosure, and the external environment can not be accurately predicted.

Disclosure uncertainty can significantly affect the stock price by influencing households' expectations. First of all, households tend to form expectations for the impacts of voluntary disclosure based on existing information. For example, after establishing the North American Free Trade Agreement (NAFTA), a large number of investors formed an expectation that more free trade zones would be established in some major districts according to the informal news that has not been officially confirmed, thus stimulating stock prices to rise for firms disclosing plans related to free trade zones substantially.

Secondly, the current market environment and regulation environment can affect households' expectations of future disclosure. When the market is very volatile or in the process of a transformation, inside traders and regular traders will release various disclosure change forecasts according to their knowledge. Thus the corresponding firms are expected to be more likely to conduct a disclosure for most investors. Then they tend to adjust their investment behavior and ultimately stimulate stock price changes.

3.1 Data

The quantitative analysis in this paper is based on three datasets. The first dataset is sourced from the Federal Reserve Economic Data (FRED), focusing on aggregate-level stock market metrics. Specifically, we utilize the monthly Fama-French Market Index data spanning from 2000 to 2023, obtained from Ken French's Data Library. The second dataset comes from the U.S. Bureau of Economic Analysis (BEA) and includes quarterly consumption data. The third dataset is from Compustat, providing quarterly earnings surprises and auditing costs.

From the model built in Section 2, τ_a and τ_c are two uncertainty parameters about voluntary disclosure in our analysis. To provide some suggestive empirical evidence, we examine the uncertainty of voluntary disclosure empirically. Following Botosan (1997), we use the volatility of the amount of voluntary disclosure in the quarterly reports of publicly traded firms across various sectors to capture the uncertainty of voluntary disclosure (UVD). To align with U.S. quarterly macroeconomic data, we aggregate firm-level UVD data to generate a U.S. country-level UVD series from 2008Q1 to 2023Q4, as illustrated in Figure 1.

3.2 Calibration

We calibrate our model using the monthly stock data, quarterly macroeconomic data and quarterly accounting data described above. Each period stands for one month. Focusing on a financial year, we set the upper bound of the interval $T = 12$ months, the month to decide whether to disclose or not is $t^* = 6$, and the instant $\Delta t = 1/365$ year. Thus there are 10 parameters need to be calibrated, which are b_1 and b_2 describing the habit formation, risk-free interest rate r_f, subjective discount rate β, relative risk aversion ϕ, expected common stock returns μ, common volatility of stock returns σ_r, variance of the time-varying uncertainty τ_ι, and two disclosure uncertainty coefficients τ_a and τ_c.

We calibrate the common expected stock returns μ using the average return of monthly Fama-French Market Index from 2000 to 2023. In terms of the common volatility of stock returns σ_r, we use the variance of monthly Fama-French Market Index for the same period. For the series of volatility varying with time ι_t, we use the the volatility index (VIX) developed by Chicago Board Options Exchange (CBOE) to measure the U.S. stock market's expectation of volatility. It is constructed using the implied volatility of a wide range of Fama-French Market Index options, which has accurately foreseen the significant fluctuations in the U.S. stock market in 2001, 2008, and 2020. Leveraging the series of monthly VIX of the U.S. Stock Market from 2000 to 2023, the variance of the time-varying volatility τ_ι is computed as the variance of VIX.

To check whether our model can fit the data very well. We also calibrate the growth rate of household consumption and its volatility. We use the growth rate of the personal consumption expenditure (PCE) from the U.S. BEA as a proxy to the growth rate of household consumption. Leveraging PCE from 2000 to 2023, we obtain the calibration of the mean and variance of the growth rate of consumption, which is denoted as e_c and v_c.

The variance of the prior distribution of voluntary disclosure variable τ_a^2 reflects households' prior knowledge about the effects of the voluntary disclosure. Borrowing from the macroeconomy literature, the impact of voluntary disclosure on the stock is measured by a multiplier(Leeper et al., 2017). In this paper, we mainly consider the multiplier effect on the return of assets (ROE) caused by voluntary disclosure. In this case, τ_a refers to the unexpected shock to the average and volatility of ROE coming from the voluntary disclosure. Thus, we first demean the ROE of firms. We set the calibration of τ_a to be 0.015. For the uncertainty of the cost of disclosure change τ_c, we use the coefficient of variation of the annual time series of UVD in the U.S. from 2000 to 2023 to calibrate it as 0.15.

Following We calibrate the risk-free interest rate r_f using the average of the Federal funds effective rate from 2000 to 2023 in the U.S.. For the subjective discount rate β, we use the equilibrium condition in the steady-state that

$$e^{-\beta} = \frac{1}{1 + r_f}$$

where the calibration of β can be obtained by $\beta = log(1 + r_f)$.

From Equation (6), we know the expectation and volatility of the growth rate of household's consumption is $Z_2 + b_2 - \frac{(Z_2+b_1)x_t}{\bar{c}_t}$ and $(1 - \frac{x_t}{\bar{c}_t})Z_1(\sigma_r + \iota_t)$. Thus, the calibration of b_1 and b_2 should satisfy the requirements of these two expressions. With the calibration of e_c and v_c listed above, we can calibrate b_1 and b_2 as 0.4 and 0.5, which is consistent with the assumption in Constantinides (1990) that b_1 and b_2 should be in $[0, 1]$. We set the relative risk-averse coefficient ϕ to be 4.3 in the U.S. The detailed description of our calibration is presented in Table 1 to visualize the probability of voluntary disclosure and its potential impact on the market value.

Given the calibration above, we can calculate the threshold of voluntary disclosure $\underline{a}(0)$ and the threshold of positive voluntary disclosure impact a^*. The impact of voluntary disclosure on market value is defined as $R(a^*) = \frac{Q_{t*}^{j,disclosure} - Q_{t*}^{j}}{Q_{t*}^{j}}$ with Q_t^j and $Q_t^{j,disclosure}$ obtained in Proposition 5. a^* is the smallest value leading to a positive voluntary disclosure impact, i.e., $R(a^*) \geq 0$. We plot four illustrative values of $\hat{a}_{t^*}$ along with the calibrated $\underline{a}(0)$ and a^* in Figure 2.

In Panel A of Figure 2, $\hat{a}_t^*$ is very low, and the probability of a firm conducting voluntary disclosure is nearly one. Since $\hat{a}_t^* < a^*$, market value rises at the announcement of a change in voluntary disclosure. Given the high probability of such a change, the price increase will be small because most of it is already priced in. In contrast, market value plunges in the unlikely event of no disclosure, which occurs if such a change imposes a huge cost on the firm.

In Panels B through D, $\hat{a}_t^* > a^*$, the market value falls if the disclosure happens. In Panel B, $a^* < \hat{a}_\tau < \underline{a}(0)$, the voluntary disclosure reduces market value even though it increases households' expected utility. Market value is lower due to higher discount rates, but the expected utility is higher due to higher expected wealth. Expected utility and market value need not move in the same direction because market value is related to marginal utility rather than the level of utility. In Panel D, $\hat{a}_t^* > 0$, indicating that the voluntary disclosure hurts market value. A voluntary disclosure is unlikely, but the market value reaction will be strongly negative if it occurs. Suppose the firm derives an unexpectedly large benefit from voluntary disclosure. In that case, it changes the status quo, which appears to work well, and market value exhibits a large drop as a result.

4 Quantitative Results

This section presents quantitative analysis to examine the effects of disclosure uncertainty on portfolio choices and stock prices from three dimensions. In the first case, we change the parameters corresponding to disclosure uncertainty to check how the possibility of voluntary disclosure and the threshold respond to the movement in parameters. We also explore how the optimal stock holdings and households' consumption behavior vary with the parameters.

Furthermore, we show the movement of the state price density, volatility, and expectation of the common stock return when parameters change. When we change specific disclosure uncertainty parameters, the other parameters are calibrated, as we state in Section 3.

4.1 The Threshold of Voluntary Disclosure

Based on the numerical simulation of Equation (8), we have the following observation describing the effects of the disclosure uncertainty on firms' disclosure choices.

Observation 1. *The threshold of the disclosure impact variable $\underline{a}(C)$ is decreasing in both the cost of voluntary disclosure C and the disclosure uncertainty τ_a.*

This observation describes features of the threshold of the disclosure impact variable, which can help us understand how the firm makes choices about whether to disclose or not. We present the results based on our numerical method in Table 2. There are two sources of disclosure uncertainty. The first one is τ_a, the second one is τ_c. From the results in Table 2, we can see when τ_a is fixed, the threshold of the disclosure impact is increasing with the uncertainty of the cost of voluntary disclosure τ_c. The economic intuition behind this result can be that when the manager needs to decide whether to conduct a voluntary disclosure, she has to take the cost associated with this move into consideration. Thus the threshold of the disclosure impact variable depends on the cost. Moreover, The high cost of disclosure is more likely to lead to a decline in firm return. Thus the firm would tend to lower the threshold to avoid a possible disclosure change. On the other hand, when τ_c is fixed, a high uncertainty of the corporate disclosure τ_a means that the firm return will decrease. So the manager tends to lower the threshold to avoid the disclosure since the effects of the disclosure are too unpredictable.

4.2 Consumption

Based on numerical simulation of Equation (6) and optimal saving path defined by Proposition 2, we can have the following Observation 2 describing the effects of the disclosure uncertainty on households' consumption and saving.

Observation 2. *The increase of the disclosure uncertainty τ_a will lower the current households' consumption and the stock holdings. At the time just before the decision instant t^*, the effects of the disclosure uncertainty on households tend to be even more significant than before.*[4]

The numerical results corresponding to Observation 2 are presented in Table 3. When the uncertainty of the cost of voluntary disclosure τ_c increases, the expected growth of households' consumption e_c, the volatility of the growth of households' consumption v_c, and the stock holdings of households $1-s$ all tend to decline. At the instant t^{*-}[5], the decline of all three variables becomes even more significant than the starting date. Based on results of Proposition 2, when the ratio of habit to consumption $\frac{x_t}{c_t}$ becomes stable, the disclosure uncertainty will lower Z_1 and Z_2, which make the effects of disclosure uncertainty on e_c, v_c, and $1-s$ even more negative. Therefore, we can conclude that due to the precautionary effects and real options effects caused by disclosure uncertainty, the households' consumption and investment in stock holdings will both become lower.

Moreover, disclosure uncertainty can also affect households' consumption through expectations. In particular, when the uncertainty of the disclosure impact τ_a increases from 0.015 to 0.055, the e_c, v_c and $1-s$ tend to be decreased by 2.12%, 1.56%, and 1.08% respectively. On the other hand, with a fixed level of disclosure uncertainty, e_c,v_c, and $1-s$ at time t^{*-} will change significantly more than the situation at time 0. This observation demonstrates that

[4]In our simulation, we obtain the paths of consumption from 0 to t^*. However, our focus in this analysis is to compare the change in households' consumption behavior between time 0 and t^*. Thus, we omit the results in between.

[5]t^{*-} stands for the left limit of t^*.

even though the disclosure uncertainty is the same at time t^{*-} and 0, the approaching to the changing date t^* can change households' expectations to a great extent. When the disclosure uncertainty is high at $\tau_a = 0.055$, this expectation effects can decrease the growth rate of households' consumption by 15.89%.Moreover, the path with lower disclosure uncertainty tends to be located higher in the graph, consistent with the findings in Table 3.

4.3 State Price Density

Based on the numerical simulation of Equation (9) and the corresponding expression of the diffusion term and the jumps, we can have the following Observation 3 describing the effects of disclosure uncertainty on State Price Density.

Observation 3. *The increase of the uncertainty of disclosure impact τ_a will increase the volatility and the magnitude of the jumps in the process of the state price density. The main reason that the state price density varies with τ_a is that when the uncertainty of disclosure impact increases, the magnitude of the jumps will become significantly bigger than before.*

The numerical simulation corresponding to Observation 3 are presented in the Panel A of Table 4. As the uncertainty of disclosure impact τ_a, and the cost of voluntary disclosure τ_c both increase, the volatility of the state price density σ_P tend to decrease. Moreover, since τ_a can affect the state price density directly, and τ_c can only exert effects through the probability of voluntary disclosure, σ_P is more sensitive to τ_a. The simulation demonstrates that when τ_c increases from 0.15 to 0.55 and τ_a is fixed at 0.015, σ_P only demonstrates an increase of 2.3%. However, when τ_a increases from 0.015 to 0.055 and τ_c is fixed at 0.15, there will be an 200% increase in σ_P. Therefore, we conclude that the uncertainty of disclosure impact τ_a rather than the uncertainty of the cost of voluntary disclosure τ_c is the crucial factor determining the state price density.

Furthermore, we move to analyze the effect on the jump in the process of state price density K_P. Because τ_c does not show up in the jump function K_P, it has no impact on the jump. On the other hand, τ_a can have a significant impact on the jump function. Based on our simulation in the Panel A of Table 4, when τ_a increases from 0.015 to 0.055 and τ_c is held at 0.15, the jump value will increase from 6.12% to 73.46%. This result demonstrates that disclosure uncertainty τ_a can make the stock value more volatile through its impact on the jump. The higher the τ_a, the more significant this effect is. Based on Proposition 3 and the simulation in Panel A of Table 4, the uncertainty of disclosure impact τ_a can be regarded as a systematic risk in the stock market. It can impact the stock price and its volatility by affecting the state price density.

4.4 Market Value

Based on the numerical simulation of Equation (10) and the corresponding expression of the diffusion term and the jumps, we can have the following Observation describing the effects of disclosure uncertainty on the market value of a stock.

Observation 4. *The increase of the uncertainty of disclosure impact τ_a will increase the volatility and the expectation of the market value. The jump in the process of the market value will be significantly bigger when disclosure uncertainty τ_a increases.*

We present the numerical simulation for Observation 4 in the Panel B of Table 4. Let us focus on the uncertainty of disclosure impact τ_a first. If we set $\tau_a = 0.015$, the disclosure uncertainty generates about a 5% risk premium with a 7.62% stock return and a 2.31% risk-free rate. However, when τ_a is set to be 0.055, the risk premium can be as high as 34%. This result can explain the risk premium puzzle to some extent since when calibrating the disclosure uncertainty correctly; our consumption-based asset pricing model can almost match the real-world risk-premium.[6]

[6]Based on the results in asset pricing literature(Epstein and Zin, 1990; Hansen and Singleton, 1982), the risk premium of the stock return usually tends to be around 7%.

Most research in this area is trying to resolve this puzzle using a long term risk model or a rare disaster model (e.g. Londono and Zhou (2017), Gabaix (2012), Rudebusch and Swanson (2008)). As for the effects of corporate disclosure on equity risk premium, Brogaard and Detzel (2015) gives strong empirical evidence that the uncertainty in corporate disclosure has significant and long-lasting real and financial implications. The distinctive feature of this paper is that we focus on the voluntary disclosure of firms instead of corporate announcements. To capture this pattern, we consider a model by adding a household sector in the framework of Pástor and Veronesi (2013), which allows the household to save and invest in stocks issued by firms simultaneously. On the other hand, the mechanism of disclosure uncertainty is very similar to a rare disaster. Both of them can affect the risk premium of the stock return through the channel of expectation. Even though the voluntary disclosure could only happen at some specific time point, the expectation about the voluntary disclosure can affect households' behavior since the very beginning. This expectation generate risks consistently, so the households need to be compensated for it. Our simulation demonstrate that the risk premium of stock returns can be mainly attributed to disclosure uncertainty when the disclosure risk is low. However, the risk premium is very sensitive to the uncertainty of disclosure impact τ_a, meaning that while τ_a increases, the risk premium will increase very quickly. Thus, this observation can be considered as a possible explanation for the "risk premium puzzle."

Furthermore, the disclosure uncertainty can not only help to explain the risk premium of stock return but also allow us to understand the movement of jump in the process of the market value of a stock. The response of the jump of market value is very similar to the jump of state price density in the last part, but the magnitude is more significant than we observe for the latter one. When $\tau_c = 0.55$, and $\tau_a = 0.055$, the change in the jump process can be as high as 320%.

The analysis about the response of the volatility of the market value to disclosure uncertainty is presented in the Panel C of Table 4. When $\tau_a = 0.015$ and $\tau_c = 0.15$, the volatility of the stock value is 14.52%. If the uncertainty of disclosure impact τ_a is set to be 0.055, and τ_c is fixed at 0.15, the volatility will turn out to be 33.69%. This numerical practice illustrates that the volatility of the market value $\sigma_{Q,t}$ is very sensitive to τ_a. However, different from what we find in the analysis of expected stock return, the effects of τ_a on $\sigma_{Q,t}$ is relatively mild. Even if we set $\tau_a = 0.055$ and $\tau_c = 0.55$, the volatility is still less than 51%, which is the real volatility of the stock market in the U.S.(Amihud et al., 2015; Shi, 2019).

The existing research on the stock market in the U.S. shows that the volatility of the stock value in the U.S. is much higher than other developed countries. The irrational behavior of investors in the U.S. could be a possible reason for the high volatility. Thus, the classical asset pricing model may not be able to fully explain the high volatility of the stock market in the U.S.. It is essential to consider the households' expectations to help us understand the high volatility of the stock market value in the U.S.. Even though disclosure uncertainty is one of the most crucial factors determining households' expectations in the U.S., we may still miss some other determinants of investors' behavior and sentiment. Besides, the jump in the volatility of market value is also very significant, especially around the disclosure date. For instance, the stock value volatility became exceptionally high in July 2015 when the U.S. Securities Regulatory Commission introduced the circuit breaker mechanism.

Due to the existence of jumps, we calculate the change of the state price density, stock return and volatility of the market value at the time t^{*+}, compared with their initial values at time 0. To further support the arguments in Observation 4, we describe the relationship between the jump variable K_Q and the disclosure uncertainty τ_a after time t^*.

Panel A of Figure 3 illustrates the unconditional expected jump risk premium $E(K_Q)$ as a function of both τ_g and τ_c, using the parameters listed in Table 1. The time length before the disclosure date is fixed at 5 months, ensuring that the uncertainty regarding $\hat{a}_{t^*}$ is fully resolved by time $t^* = 5$, given that $\hat{a}_{t^*}$. The figure demonstrates that $E(K_Q)$ rises with increases in τ_a and decreases in τ_c. In the baseline scenario, where τ_a is 0.02 and τ_c is 0.15, $E(K_Q)$ is 5.1 basis points; however, when τ_a is elevated to 0.01, $E(K_Q)$ reaches 5.5 basis points.

Panels B and C of Figure 3 illustrate the conditional expected jumps $E(K_{Q,Disclosure})$ and $E(K_{Q,noDisclosure})$, respectively, computed similarly to $E(K_Q)$ by integrating out the uncertainty regarding $\hat{a}_{t^*}$. Panel B presents $E(K_{Q,Disclosure})$, corresponding to the Expected Announcement Return (EAR) discussed in the asset pricing literature. Panel C shows that $E(K_{Q,noDisclosure})$ is positive and increases with both τ_a and τ_c, though it remains smaller in magnitude compared to $E(K_{Q,Disclosure})$. This difference arises because the probability-weighted average of the jumps in Panels B and C is nearly zero (as shown in Panel A), and the unconditional probability of a voluntary disclosure is below 0.5. This probability, defined by Equation (8) and assessed during the period preceding the disclosure date t^*, ranges from 11% to 49%, depending on the values of τ_a and τ_c (it is 29% in the baseline scenario of $\tau_a = 0.015$ and $\tau_c = 0.15$).

Theoretically speaking, the higher the return, the less possible a disclosure will happen. In other words, investors will most likely have very similar expectations about the future stock market. Thus, the jump K_Q is strictly positively correlated with disclosure uncertainty. However, in this situation, due to the less sensitivity to the disclosure, the magnitude of the jump is significantly lower than we observe in the low return case. On the other hand, when the return is low, it is highly possible that the manager will conduct a voluntary disclosure. Nevertheless, the households' expectations about how the voluntary disclosure can differ from each other. For example, during the financial crisis, the return of the stock market is very low. Some investors expect that the firms will make more announcements about new projects, but others may think the firms should stay silent and wait for a better timing. So, the jump in the stock value can be affected by two mechanisms working against each other. Moreover, the jump can also be attributed to over-shooting. When a voluntary disclosure is in line with most households' expectations, the optimism of the households will generate more investment driving up the stock value. While the disclosure is not what most investors expected, the stock value can decline because of the pessimism of investors.

4.5 Expected Disclosure Impact

To check whether our model can match the data, we conduct a dynamic analysis based on the disclosure variables' simulated paths and the stock market patterns using the calibrated parameters in Table 1. In the first case, based on Equation (3) and the prior distribution of the disclosure variable, we simulate the expected disclosure impact $\hat{a}_t$, disclosure impact a_t, and the threshold of voluntary disclosure $\underline{a}(C)$ for 10000 times, and average across the simulated paths. Then we have the three paths corresponding to the three variables.

In Figure 4, we present the dynamics of expected returns with and without voluntary disclosure based on the simulated series of $\hat{a}_t$ and $\underline{a}(C)$. From Panel A to Panel B, we plot the estimated series when the common stock return is set to be 0.01 and 0.6. At the instant t^{*-}, the firm can compare the threshold of the expected return with the expectation to decide whether it is necessary to disclose. In Panel A and Panel B, the firm will carry out voluntary disclosure when the threshold is higher than the expected return at the instant t^*. Thus, there is a jump in expected return.

We set the common stock return as 0.01 to capture a recession in Panel A. We observe that, on average, voluntary disclosure happens since the posterior mean of the return is lower than the threshold at t^*. The jump is positive, meaning the expected return after t^* is higher than before. This result corresponds to the situation in which voluntary disclosure positively affects expected return, showing that the households tend to think that voluntary disclosure could be good for the firm. Thus, voluntary disclosure is beneficial for households during a recession. Meanwhile, we simulate the scenario of a boom by setting $\mu = 0.6$ in Panel B. It shows that, on average, the firm does not conduct voluntary disclosure because its posterior mean of the return is higher than the threshold. Then, the expected return afterward will be kept at the original level after the instant t^*. Therefore, it might be better for households if firms reduce voluntary disclosure during a boom.

Focusing on the paths with voluntary disclosure using the calibration in Table 1, the series of state price density, stock return, and volatility of the market value is simulated based on Equation (9), (10), and the related expressions of the volatility and drifts. We also change the uncertainty disclosure impact τ_a when generating the series. The simulated paths are presented in Figure 5. The dynamics demonstrate that during the interval $[0, T]$, if the disclosure uncertainty becomes higher, the state price density, stock return, and the volatility of market value all tend to be driven up, which is consistent with what we find in the comparative static analysis. Meanwhile, due to the effects of the learning process, in the interval $[0, t^{*-}]$ and $[t^{*+}, T]$ all three variables exhibit to decreasing. Moreover, after the instant t^*, because the expectation of voluntary disclosure does not exist anymore, the decrease is significantly bigger than when there is disclosure uncertainty.

Based on calibration in Table 1, we set the uncertainty of disclosure impact τ_a to be 0.03. From the simulated paths of the risk premium and market value volatility, we find that the average risk premium during the disclosure interval is 4.38%, which is slightly lower than the risk premium in the U.S.. This result demonstrates that adding the disclosure uncertainty can help explain the equity risk premium puzzle in the U.S.. On the other hand, the average volatility of market value is about 12.34%, which is significantly lower than the real value in the U.S. stock market. This observation illustrates that the U.S.'s market's high volatility can only be partially attributed to disclosure uncertainty. Besides the factors addressed in this paper, investor sentiment can also be a crucial driving force for the fluctuations in the stock market.

Additionally, the underestimation of jump risk in our model could also lower the volatility. Because of this paper's setting, the voluntary disclosure can only happen at time t^*. Thus the stock return can only jump at the instant t^*. However, in the real world, the date of voluntary disclosure is not fixed, and the firm might disclose more frequently than we assume. These missing components could also pull down the simulated volatility in the numerical results.

We relax the assumption that t^* is exogenously given. Instead, we allow t^* to be optimally chosen based on the model dynamics. We solve the model numerically, presenting the results in Figure 6, which illustrates the dynamics of the market-to-book ratio (M/B) and stock return volatility for firms with and without voluntary disclosure. Figure 6 shows the average paths of M/B and volatility with and without voluntary disclosure, t^*, is determined endogenously. Depending on the return path, voluntary disclosure can occur at any point between $t = 6$ and $t = 12$ rather than at $t = 6$ for sure. The left figures in both panels report averages across simulations where the optimal t^* falls between Years 6 and 8.

Our findings indicate that allowing t^* to be optimally chosen does not alter the main conclusions in the exogenous benchmark. The M/B exhibits a rise and fall pattern during the post-periods of voluntary disclosure, while stock return volatility spikes around the optimal voluntary disclosure time, reflecting heightened market sensitivity. And as the disclosure uncertainty τ_a increases, the spike becomes more significant. These results underscore the importance of strategic disclosure timing in influencing key market metrics.

5 Robustness to Misspecification

This section conducts further sensitivity tests by changing the habit formation setting, relative risk aversion, and stock holding share, which are three variables closely related to stock price. The consistent results obtained in the following analysis can be used to show the robustness of the effects of disclosure uncertainty on stock return and the volatility of stock market value.

5.1 Habit Formation

In the benchmark analysis presented above, we introduce habit formation in the household's utility function, as suggested by Constantinides (1990). This setting is beneficial in the calibration of the parameters related to consumption and the risk-free return. Constantinides (1990) claims that without habit formation, the real world risk-free return can only make sense in the asset pricing model when the relative risk aversion is low. When the ratio of habit formation is increasing, the calibrated risk-free return can be adapted to a broader range of relative risk aversion rate. In this paper, we also come up with a similar conclusion. By introducing the habit formation using the real consumption data, we can maintain the risk-free return and relative risk aversion in a reasonable range. Because the parameters related to consumption is not included in the asset pricing equation, we change the ratio of habit formation set in the last part, which may be against the conditions satisfied with the real consumption data. However, this sensitivity test can help us examine whether the variables related to stocks exhibit a similar response to disclosure uncertainty under different settings about habit formation.

The numerical results of the sensitivity test on habit formation and disclosure uncertainty are presented in the following tables. Panel A, Panel B, and Panel C of Table 5 correspond to the results of the state price density, stock return, and the volatility of stock market value, respectively. In particular, because of the setting in our model, all three variables exhibit similar features. When the uncertainty of disclosure impact τ_a increases, each of them will go up. Furthermore, there is a jump for each of the three variables at the instant t^{*+}. Besides, the magnitude of the jump is also increasing in τ_a. Besides, we present the quantitative analysis of the discussion above in Table 5.

For the habit formation, when the uncertainty of disclosure impact τ_a is relatively low, the state price density, stock return, and volatility of market value are all decreasing in habit formation. However, when the disclosure uncertainty τ_a becomes higher, the three variables above will turn out to be increasing in habit formation. This result demonstrates that households are more likely to maintain their consumption level when disclosure uncertainty is low. Therefore, the stock return asked by the households with a high ratio of habit formation is low. On the other hand, in a scenario with high τ_a, we can observe more fluctuations in household consumption. Because in this case, a high ratio of habit formation means the household will ask for a higher stock return. Finally, the numerical results in Panel A show that the magnitude of the jump will not respond significantly to the change in habit formation.

For the ratio of habit to consumption h_t, when the stationary value $\bar{h}$ is equal to 0, it means that there is no habit formation in the steady-state. In other words, we rule out the setting related to habit formation. However, even when the habit formation is excluded, the effects of disclosure uncertainty on the stock return and the volatility of stock market value are still significant. This observation shows that our benchmark analysis is robust to the setting of households' habit formation.

5.2 Relative Risk Aversion

The relative risk aversion reflects the households' attitude to the risk in the stock market. In the sensitivity test conducted in the current section, we will allow the relative risk aversion to move in a broader range. Therefore, we can examine the benchmark results' sensitivity about the state price density, stock return, and volatility of stock market value.

The numerical results of the sensitivity test are presented in the following tables. Panel A, Panel B and Panel C of Table 6 correspond to the results of the state price density, stock return, and the volatility of stock market value, respectively. In the first case, all three variables are increasing in the uncertainty of disclosure impact τ_a across different calibrations of relative risk aversion we propose. Unlike what we observe in habit formation experiments above, state price density, stock return, and stock market volatility are very sensitive to the change in the relative risk aversion. In particular, this sensitivity's effects can be combined with the effects caused by high τ_a). When τ_a is high, the positive effect of increasing relative risk aversion on risk premia will be higher than the case when the disclosure uncertainty is relatively low. Meanwhile, these results also demonstrate that the relative risk aversion can not reflect the investor's genuine attitude towards the risk since the three stock variables' response to changes in relative risk aversion can be quite different when the uncertainty disclosure impact τ_a varies. Moreover, the quantitative analysis above is presented in Table 6.

5.3 Stock Holding Share

Because the stock holding of the household is the only source of the internal finance of firms in our model, the analysis of stock holding share of the household is crucial to understand the relationship between household consumption and stock return of firms. In the following sensitivity test, we will adopt different stock holding shares of households in the numerical analysis to explore the effects of internal finance on the stock market performance of firms under different disclosure uncertainty.

The numerical results of the sensitivity test on the stock holding share are presented in the following tables. Panel A, Panel B and Panel C of Table 7 correspond to the simulation results of the state price density, stock return, and the volatility of stock market value, respectively. Consistent with what we find in the test on habit formation and relative risk aversion, all three stock performance variables are still positively correlated with disclosure uncertainty τ_a. Besides, when the disclosure uncertainty is relatively low, the three variables all exhibit to increase in the stock holding share. However, this relation is no longer true when disclosure uncertainty τ_a becomes higher. For example, in our numerical experiments, when the disclosure uncertainty τ_a is higher than 0.45, both the state price density and stock return will turn out to be decreasing in the stock holding share of a household.

Moreover, stock market value volatility will start decreasing in the stock holding share of households if the disclosure uncertainty τ_a is bigger than 0.35. These results can be attributed to investors' patterns in the stock market of the U.S. because most of the stock market investors in the U.S. are individual investors. They are more likely to participate in stock trading only when uncertainty is low since they are more risk-averse and far less professional than institutional investors in most cases (e.g. Kaniel et al. (2008), Kaniel et al. (2012), Gompers and Metrick (2001)). In particular, when disclosure uncertainty is low, the higher participation rate of the individual investors can generate more returns. The accompanying high trade volumes in the stock market can also lead to high stock market value volatility. On the other hand, fewer individual investors will show up in the stock market when disclosure uncertainty τ_a becomes higher. From the household's perspective, holding more stocks issued by firms can only generate fewer returns than holding risk-free assets in this case. Thus they will trade less in the stock market than when disclosure uncertainty is low, leading to lower stock volatility. The quantitative results discussed above are presented in Table 7.

6 Conclusion

We develop a dynamic asset pricing model by adding a firm's voluntary disclosure based on the framework proposed by Pástor and Veronesi (2013). Our model examines the relationship between disclosure uncertainty, household consumption, and stock market dynamics.

The model gives us a system of equations featuring the optimal household consumption and stock holdings. Then, we calibrate our model with the stock market data in the U.S. . Our simulation gives us the dynamics of household consumption when disclosure uncertainty changes. Our numerical results show that household consumption and stock holdings both decrease in the disclosure uncertainty. Before the disclosure change moment t^*, households will pay more attention to the disclosure uncertainty because of the expected effects. Thus, there is a jump in the effects of disclosure uncertainty at this moment.

The firm chooses whether to disclose by comparing the effects of disclosing more with sticking to the current state. Disclosure willy only happen when it is sufficiently beneficial, which is mainly dependent on the cost of disclosure change c. Furthermore, the threshold of a_t decreases in disclosure uncertainty and the cost of disclosure change. We also find the jump caused by the disclosure change is bigger during the recession than the jump during the boom, implying that the disclosure change tends to be more effective in the recession than in the boom.

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Figure 1. Disclosure Uncertainty of the U.S.

This figure plots the disclosure uncertainty measured by the number of items actually and voluntarily disclosed by U.S. firms divided by the total number of relevant items that should be disclosed from 2008Q1 to 2023Q4.

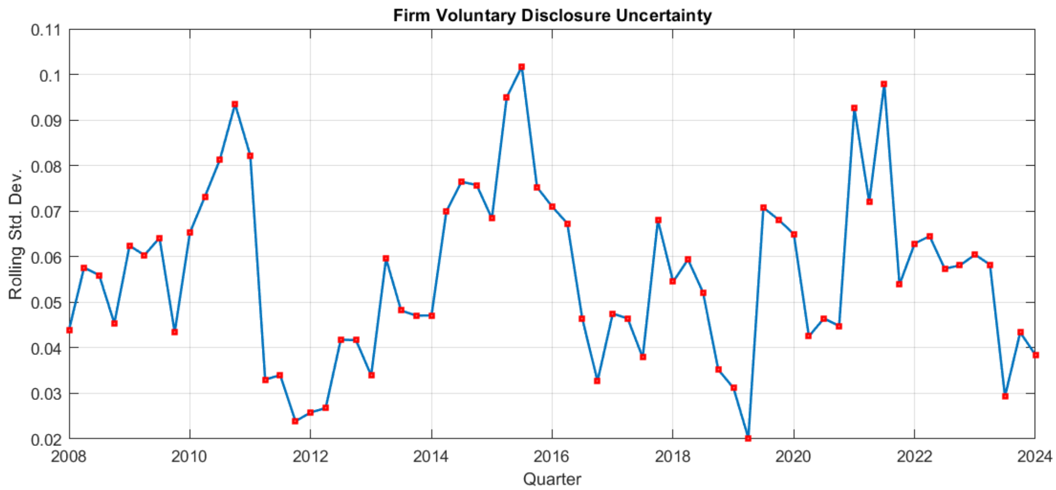


Figure 2. Probability of Voluntary Disclosure

The shaded area represents the probability of a voluntary disclosure, as perceived by shareholders just before time t^*. The bell curve represents the normal distribution of the random threshold $\underline{a}(C)$. The four panels illustrate four possible examples of the posterior mean $\hat{a}_t^*$ relative to a^*, $\underline{a}(0)$, and zero. The vertical dotted lines are drawn at a^*, $\underline{a}(0)$, and zero. The normal distribution as well as the values of a^* and $\underline{a}(0)$ are computed based on the parameter values in Table 1.

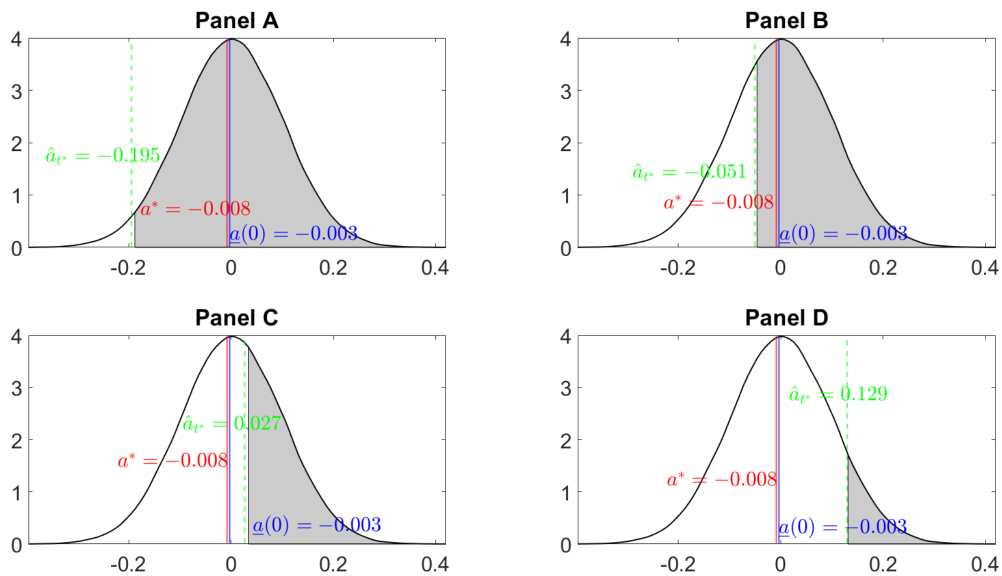
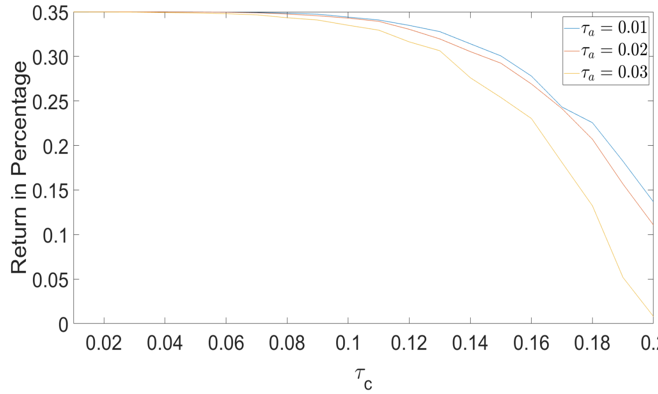


Figure 3. The Jump in Return around the Disclosure Date

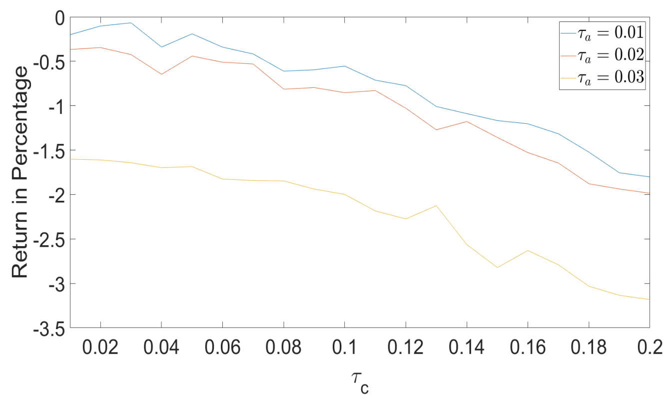
Panel A plots the expected jump risk premium $E(K_Q)$ as a function of τ_a and τ_c. Panel B presents $E(K_{Q,Disclosure})$, showing the conditional expected jump with voluntary disclosure. Panel C illustrates $E(K_{Q,noDisclosure})$, depicting the conditional expected jump without voluntary disclosure. The length of periods before the disclosure date is set to be 6 months, ensuring uncertainty regarding $\hat{a}_{t^*}$ is integrated out as shareholders perceived at the beginning of the downturn. All other parameters are the same in Table 1.

Panel A: Unconditional Expected Jump



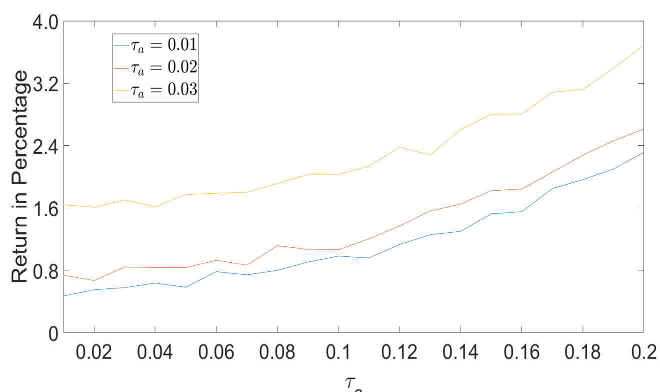
3a.png

Panel B: Conditional Expected Jump with Voluntary Disclosure



3b.png

Panel C: Conditional Expected Jump without Voluntary Disclosure

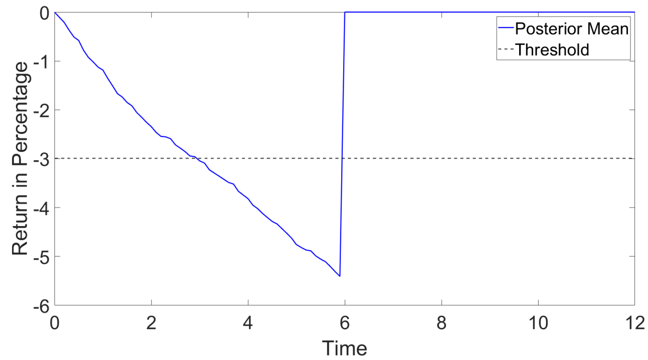


3c.png

Figure 4. The Impact of Voluntary Disclosure around the Disclosure Date

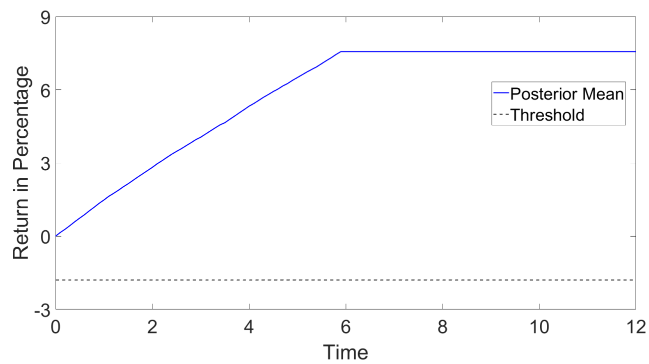
This figure plots the expected dynamics of the disclosure impact (solid line) under different levels of common stock returns. Panel A depicts the scenario with a low common stock return ($\mu = 0.01$), while Panel B shows the scenario with a high common stock return ($\mu = 0.6$). Both panels report averages across simulations where voluntary disclosure could occur at $t^* = 6$, and the whole disclosure interval lasts for $T = 12$ periods. The dotted line represents the simulated threshold $\underline{a}(c)$. Both two lines are average paths across 10000 simulated samples.

Panel A: Low Market Return(Recession)



4a.png

Panel B: High Market Return(Boom)

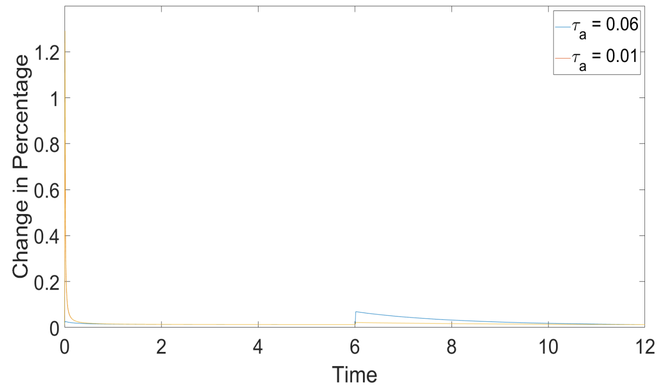


4b.png

Figure 5. The Dynamics of Stock Market Variables

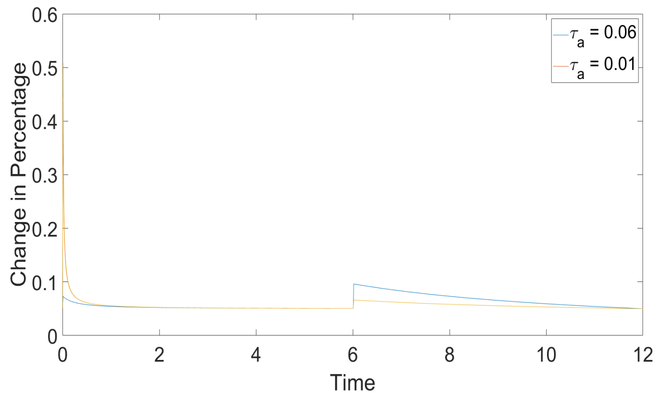
This figure plots the change in the percentage of stock return (Panel A), stock volatility (Panel B), and state price density (Panel C) around the disclosure date over time under different levels of disclosure uncertainty ($\tau_a = 0.06$, $\tau_a = 0.01$). All parameters follow the calibration in Table 1.

Panel A: Stock Return



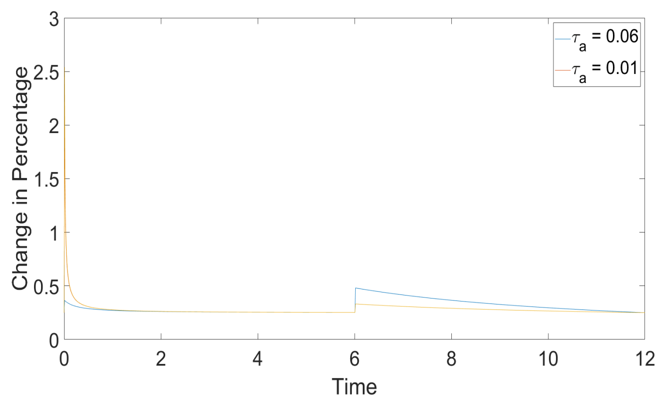
5 Stock Return.png

Panel B: Stock Volatility



5 Stock Volatility.png

Panel C: State Price Density



5 State price density.png

Figure 6. Endogenous Disclosure Timing

This figure shows the dynamics of the market-to-book ratio (M/B) and stock return volatility with and without voluntary disclosure when the disclosure time is endogenous. Each subfigure presents two lines: the solid line corresponds to the case of high disclosure uncertainty ($\tau_a = 0.06$), and the dashed line captures the scenario of low disclosure uncertainty($\tau_a = 0.01$).

6.png

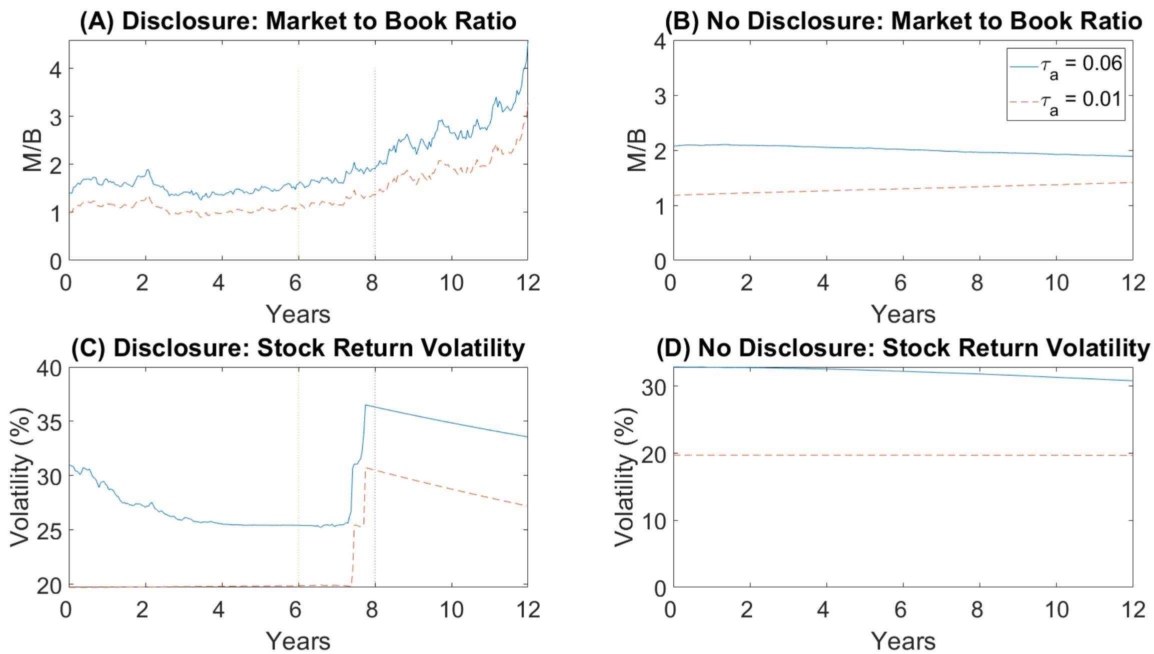


Table 1. Calibration

This table reports the calibration results described in Section 3. The first column shows the symbols of the parameters we use in the paper. The second column describes the corresponding symbol. The third column presents the calibrated value of each parameter. All the data sources are listed at the beginning of Section 3. The expected common stock returns, common volatility of stock returns, growth rate of shareholder consumption, and variance of the growth rate of shareholder consumption correspond to monthly data and are expressed in percentages.

Symbol	Description	Value
Stock Return		
μ_r	Expected common stock returns	0.100
σ_r	Common volatility of stock returns	3.380
τ_ι	Variance of the time-varying volatility returns	7.558
Shareholder Consumption		
e_c	Growth rate of shareholder consumption	0.773
v_c	Variance of the growth rate of shareholder consumption	0.667
Disclosure Uncertainty		
τ_a	Standard deviation of disclosure variable	0.025
τ_c	Standard deviation of cost variable	0.15
Habit Formation		
b_1	Habit parameter 1	0.4
b_2	Habit parameter 2	0.5
Other Parameters		
β	Subjective discount rate	0.070
r_f	Risk-free interest rate	0.174
ϕ	Relative risk aversion	4.159

Table 2. Disclosure Uncertainty and the Threshold of Corporate Disclosure Variable

This table reports the numerical results of the threshold of the corporate disclosure variable. The first column presents the different τ_a used for this numerical analysis, and the first row corresponds to the values of τ_c. Each entry in the middle of this table stands for the threshold of disclosure variable corresponding to the specific value of τ_a and τ_c.

	$\underline{a}(C)$				
Uncertainty	τ_c=0.15	τ_c=0.25	τ_c=0.35	τ_c=0.45	τ_c=0.55
τ_a=0.015	0.0001	0.0009	0.0016	0.0028	0.0049
τ_a=0.025	-0.0004	0.0007	0.0018	0.0034	0.0059
τ_a=0.035	-0.0026	-0.0016	0.0004	0.0025	0.0051
τ_a=0.045	-0.0081	-0.0067	-0.0059	-0.0026	0.0008
τ_a=0.055	-0.0269	-0.0206	-0.0198	-0.0176	-0.0123

Table 3. Disclosure Uncertainty and the Shareholder Choices

This table reports the numerical results of the variables corresponding to shareholder consumption. The first column presents the different τ_a used for this numerical analysis. The columns (1)–(3) correspond to the numerical values of e_c, v_c, and s at time 0. The Columns (4)–(6) correspond to the numerical values of e_c, v_c, and s at time t^{*-}. The columns (7)–(9) show the change of e_c, v_c, and s between time 0 and time t^{*-} in ratio.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Time 0			Time t^{*-}			Change		
τ_a	e_c	v_c	$1-s$	e_c	v_c	$1-s$	e_c	v_c	$1-s$
0.015	0.0398	0.0328	0.8961	0.0372	0.0297	0.8856	0.0016	0.0031	0.0105
0.025	0.0384	0.0315	0.8856	0.0351	0.0276	0.8659	0.0033	0.0039	0.0197
0.035	0.0375	0.0308	0.8821	0.0321	0.0246	0.8521	0.0054	0.0062	0.0300
0.045	0.0351	0.0296	0.8795	0.0287	0.0227	0.8332	0.0064	0.0019	0.0463
0.055	0.0346	0.0289	0.8654	0.0261	0.0208	0.8126	0.0085	0.0081	0.0528

Table 4. Disclosure Uncertainty and the State Price Density

This table reports the numerical results of the variables corresponding to the state price density when disclosure uncertainty changes. Panel A reports the values of σ_P at $t = 0$ and $t = t^{*+}$. The top sub panel shows the volatility of state price density at time 0. The bottom sub panel represents the Jump K_P at time t^{*+} -the time slightly after t^*- for different disclosure uncertainty settings τ_a and τ_c. Panel B reports the values of μ_Q at at $t = 0$ and $t = t^{*+}$ The first column presents the different τ_a used for this numerical analysis, and the first row corresponds to the values of τ_c. Each entry in the top sub panel stands for the stock return at time 0, $\mu_{Q,0}$ corresponding to the specific value of τ_a and τ_c. The results of the Jump after t^* corresponding to different disclosure uncertainty settings, K_Q at time t^{*+} are presented in the bottom sub panel. Panel C reports the values of σ_Q at at $t = 0$ and $t = t^{*+}$. The first column presents the different τ_a used for this numerical analysis, and the first row corresponds to the values of τ_c. Each entry in the top panel stands for the volatility of market value at time 0, $\sigma_{Q,0}$ corresponding to the specific value of τ_a and τ_c. The results of the Jump after t^* corresponding to different disclosure uncertainty settings, K_Q at time t^{*+} are presented in the bottom panel.

Panel A: State Price Density										
	$\sigma_{P,t}$ at $t = 0$					Jump at t^{*+}				
	τ_c=0.15	0.25	0.35	0.45	0.55	τ_c=0.15	0.25	0.35	0.45	0.55
τ_a=0.015	0.4569	0.4583	0.4639	0.4687	0.4712	0.063	0.062	0.064	0.059	0.068
0.025	0.5128	0.5145	0.5198	0.5203	0.5267	0.214	0.226	0.231	0.227	0.236
0.035	0.6589	0.6685	0.6703	0.6731	0.6798	0.412	0.436	0.428	0.439	0.417
0.045	0.9026	0.9087	0.9123	0.9164	0.9196	0.582	0.596	0.573	0.602	0.579
0.055	1.1287	1.1385	1.1462	1.1498	1.1523	0.786	0.795	0.813	0.764	0.809

Panel B: Stock Return										
	$\mu_{Q,t}$ at $t = 0$					Jump at t^{*+}				
	τ_c=0.15	0.25	0.35	0.45	0.55	τ_c=0.15	0.25	0.35	0.45	0.55
τ_a=0.015	0.0623	0.0628	0.0631	0.0634	0.0635	0.2324	0.2869	0.2654	0.2463	0.2387
0.025	0.0762	0.0776	0.0784	0.0786	0.0791	0.6893	0.6952	0.7064	0.7168	0.7293
0.035	0.1208	0.1267	0.1289	0.1198	0.1239	0.412	0.436	0.428	0.439	0.417
0.045	0.2169	0.2178	0.2183	0.2196	0.2208	1.582	1.631	1.698	1.706	1.718
0.055	0.3682	0.3697	0.3703	0.3721	0.3734	2.036	2.087	2.123	2.197	2.284

Panel C: Volatility of Market Value										
	$\sigma_{Q,t}$ at $t = 0$					Jump at t^{*+}				
	τ_c=0.15	0.25	0.35	0.45	0.55	τ_c=0.15	0.25	0.35	0.45	0.55
τ_a=0.015	0.1452	0.1468	0.1473	0.1489	0.1496	0.1563	0.1632	0.1596	0.1587	0.1543
0.025	0.1625	0.1637	0.1648	0.1679	0.1693	0.3298	0.3321	0.3368	0.3394	0.3376
0.035	0.2019	0.2035	0.2046	0.2059	0.2098	0.5697	0.5703	0.5716	0.5732	0.5741
0.045	0.2687	0.2692	0.2703	0.2709	0.2712	0.7236	0.7368	0.7421	0.7698	0.7712
0.055	0.3369	0.3387	0.3396	0.3398	0.3389	0.8654	0.8761	0.8894	0.8903	0.8912

Table 5. Disclosure Uncertainty, Habit Formation and State Price Density

This table reports the numerical results of the variables corresponding to the stock return when habit formation varies. Panel A reports the values of σ_P at $t = 0$ and $t = t^{*+}$. The first column presents the different τ_a used for this numerical analysis, and the first row corresponds to the values of $\bar{h}$. Each entry in the top panel for the state price density at time 0, $\sigma_{P,0}$ corresponding to the specific value of τ_a and $\bar{h}$. The results of the $\sigma_{P,t}$ just after t^* corresponding to different values of τ_a and $\bar{h}$ at time t^{*+} are presented in the bottom panel. Panel B reports the values of μ_Q at at $t = 0$ and $t = t^{*+}$. The first column presents the different τ_a used for this numerical analysis, and the first row corresponds to the values of $\bar{h}$. Each entry in the top panel for the state price density at time 0, $\mu_{Q,0}$ corresponding to the specific value of τ_a and $\bar{h}$. The results of the $\mu_{Q,t}$ just after t^* corresponding to different values of τ_a and $\bar{h}$ at time t^{*+} are presented in the bottom panel. Panel C reports the values of σ_Q at at $t = 0$ and $t = t^{*+}$. The first column presents the different τ_a used for this numerical analysis, and the first row corresponds to the values of $\bar{h}$. Each entry in the top panel stands for the state price density at time 0, $\sigma_{Q,0}$ corresponding to the specific value of τ_a and $\bar{h}$. The results of the $\sigma_{Q,t}$ just after t^* corresponding to different values of τ_a and $\bar{h}$ at time t^{*+} are presented in the bottom panel.

Panel A: State Price Density

	$\sigma_{P,t}$ at $t = 0$					$\sigma_{P,t}$ at t^{*+}				
	$\bar{h}$=0	0.1	0.3	0.5	0.8	$\bar{h}$=0	0.1	0.3	0.5	0.8
τ_a=0.015	0.5214	0.5129	0.5018	0.4521	0.4085	0.5369	0.5317	0.5298	0.5231	0.5164
0.025	0.5698	0.5621	0.5594	0.5432	0.5083	0.5893	0.5865	0.5791	0.5706	0.5188
0.035	0.6973	0.6928	0.6836	0.6471	0.6237	0.7968	0.7861	0.7846	0.7831	0.7726
0.045	0.8567	0.8514	0.8362	0.8216	0.8369	1.2698	1.2759	1.3064	1.3982	1.4692
0.055	1.0756	1.0985	1.1643	1.1267	1.2084	1.6329	1.6954	1.7028	1.7163	1.8957

Panel B: Stock Return

	$\mu_{Q,t}$ at t=0					$\mu_{Q,t}$ at t^{*+}				
	$\bar{h}$=0	0.1	0.3	0.5	0.8	$\bar{h}$=0	0.1	0.3	0.5	0.8
τ_a=0.015	0.1023	0.0986	0.0913	0.0854	0.0731	0.1289	0.1274	0.1167	0.1132	0.0865
0.025	0.1354	0.1308	0.1246	0.1097	0.0952	0.1528	0.1513	0.1498	0.1132	0.1087
0.035	0.1567	0.1524	0.1489	0.1396	0.1127	0.2386	0.2219	0.2164	0.2237	0.2314
0.045	0.2248	0.2169	0.2087	0.2289	0.2374	0.3574	0.3698	0.3796	0.3847	0.3908
0.055	0.3287	0.3318	0.3397	0.3457	0.3482	0.5873	0.5964	0.6082	0.6034	0.64261

Panel C: Volatility of Market Value

	$\sigma_{Q,t}$ at t=0					$\sigma_{Q,t}$ at t^{*+}				
	$\bar{h}$=0	0.1	0.3	0.5	0.8	$\bar{h}$=0	0.1	0.3	0.5	0.8
τ_a=0.015	0.1862	0.1854	0.1763	0.1629	0.1158	0.2095	0.2017	0.1956	0.1873	0.1438
0.025	0.2157	0.2134	0.2118	0.2074	0.1967	0.2698	0.2615	0.2578	0.2564	0.2367
0.035	0.2379	0.2316	0.2268	0.2153	0.2284	0.3246	0.3207	0.3198	0.3243	0.3317
0.045	0.3087	0.3012	0.3034	0.3128	0.3169	0.3984	0.4028	0.4097	0.4183	0.4579
0.055	0.4139	0.4287	0.4396	0.4692	0.4926	0.5618	0.5931	0.6082	0.6137	0.6483

Table 6. Disclosure Uncertainty, Relative Risk Aversion and State Price Density

This table reports the numerical results of the variables corresponding to the stock return when relative risk aversion varies. Panel A reports the values of σ_P at $t = 0$ and $t = t^{*+}$. The first column presents the different τ_a used for this numerical analysis, and the first row corresponds to the values of ϕ. Each entry in the top panel stands for the state price density at time 0, $\sigma_{P,0}$ corresponding to the specific value of τ_a and ϕ. The results of the $\sigma_{P,t}$ just after t^* corresponding to different values of τ_a and ϕ at time t^{*+} are presented in the bottom panel. Panel B reports the values of μ_Q at at $t = 0$ and $t = t^{*+}$. The first column presents the different τ_a used for this numerical analysis, and the first row corresponds to the values of ϕ. The top panel stands for the state price density at time 0, $\mu_{Q,0}$ corresponding to the specific value of τ_a and ϕ. The results of the $\mu_{Q,t}$ just after t^* corresponding to different values of τ_a and ϕ at time t^{*+} are presented in the bottom panel. Panel C reports the values of σ_Q at at $t = 0$ and $t = t^{*+}$. The first column presents the different τ_a used for this numerical analysis, and the first row corresponds to the values of ϕ. The top panel stands for the state price density at time 0, $\sigma_{Q,0}$ corresponding to the specific value of τ_a and ϕ. The results of the $\sigma_{Q,t}$ just after t^* corresponding to different values of τ_a and ϕ at time t^{*+} are presented in the bottom panel.

Panel A: State Price Density

	$\sigma_{P,t}$ at t=0					$\sigma_{P,t}$ at t^{*+}				
	ϕ=1	2	3	4	5	ϕ=1	2	3	4	5
τ_a=0.015	0.4325	0.4482	0.4956	0.5864	0.6675	0.4967	0.5013	0.5028	0.5569	0.6834
0.025	0.4612	0.4859	0.5493	0.6945	0.7861	0.5149	0.6028	0.6493	0.7025	0.8564
0.035	0.4287	0.6592	0.7368	0.8456	0.9836	0.5296	0.6238	0.7698	0.8547	1.7726
0.045	0.5987	0.8294	0.9638	1.6328	1.8524	0.6354	1.2578	2.6381	3.6284	4.5283
0.055	0.6237	1.0854	1.1239	1.9597	2.2084	0.6568	1.8527	2.9653	6.5682	9.3286

Panel B: Stock Return

	$\mu_{Q,t}$ at t=0					$\mu_{Q,t}$ at t^{*+}				
	ϕ=1	2	3	4	5	ϕ=1	2	3	4	5
τ_a=0.015	0.0956	0.0932	0.0854	0.0823	0.0796	0.1086	0.1032	0.0951	0.0926	0.0843
0.025	0.1027	0.0965	0.0912	0.0867	0.0809	0.1237	0.1219	0.1364	0.1467	0.1598
0.035	0.1269	0.1203	0.1168	0.1052	0.1035	0.1954	0.2146	0.3862	0.5367	0.6982
0.045	0.1843	0.1823	0.1746	0.1789	0.1854	0.2936	0.3452	0.4891	0.8634	0.9678
0.055	0.2467	0.2581	0.5329	0.6938	0.7954	0.3975	0.4821	0.6453	0.8961	0.9937

Panel C: Volatility of Market Value

	$\sigma_{Q,t}$ at t=0					$\sigma_{Q,t}$ at t^{*+}				
	ϕ=1	2	3	4	5	ϕ=1	2	3	4	5
τ_a=0.015	0.2364	0.1956	0.1368	0.1012	0.0934	0.2596	0.2507	0.2413	0.1963	0.1754
0.025	0.2467	0.2023	0.1964	0.1623	0.1129	0.2874	0.2743	0.2708	0.2859	0.3051
0.035	0.2598	0.2139	0.2048	0.2182	0.2467	0.3019	0.3387	0.3956	0.4028	0.4273
0.045	0.2931	0.3287	0.3369	0.3516	0.3728	0.3549	0.4128	0.5397	0.6482	0.7396
0.055	0.3362	0.3896	0.4027	0.4864	0.5519	0.4012	0.4683	0.5674	0.5918	0.8564

Table 7. Disclosure Uncertainty, Stock Holding Share and State Price Density

This table reports the numerical results of the variables corresponding to the stock return when stock holding share varies. Panel A reports the values of σ_P at $t = 0$ and $t = t^{*+}$. The first column presents the different τ_a used for this numerical analysis, and the first row corresponds to the values of $\bar{s}$. The top panel stands for the state price density at time 0, $\sigma_{P,0}$ corresponding to the specific value of τ_a and $\bar{s}$. The results of the $\sigma_{P,t}$ just after t^* corresponding to different values of τ_a and $\bar{s}$ at time t^{*+} are presented in the bottom panel. Panel B reports the values of μ_Q at at $t = 0$ and $t = t^{*+}$. The first column presents the different τ_a used for this numerical analysis, and the first row corresponds to the values of $\bar{s}$. The top panel stands for the state price density at time 0, $\mu_{Q,0}$ corresponding to the specific value of τ_a and $\bar{s}$. The results of the $\mu_{Q,t}$ just after t^* corresponding to different values of τ_a and $\bar{s}$ at time t^{*+} are presented in the bottom panel. Panel C reports the values of σ_Q at at $t = 0$ and $t = t^{*+}$. The first column presents the different τ_a used for this numerical analysis, and the first row corresponds to the values of $\bar{s}$. The top panel stands for the state price density at time 0, $\sigma_{Q,0}$ corresponding to the specific value of τ_a and $\bar{s}$. The results of the $\sigma_{Q,t}$ just after t^* corresponding to different values of τ_a and $\bar{s}$ at time t^{*+} are presented in the bottom panel.

Panel A: State Price Density

	$\sigma_{P,t}$ at t=0					$\sigma_{P,t}$ at t^{*+}				
	$\bar{s}$=0.9	0.8	0.7	0.6	0.5	$\bar{s}$=0.9	0.8	0.7	0.6	0.5
τ_a=0.015	0.2015	0.2269	0.2587	0.3059	0.3267	0.2436	0.2874	0.3128	0.3761	0.4295
0.025	0.4269	0.4175	0.4029	0.4346	0.4973	0.5084	0.5391	0.5238	0.5869	0.6243
0.035	0.8594	0.7561	0.6923	0.6687	0.7213	1.3846	1.2769	1.1634	1.0853	1.3672
0.045	1.2856	1.0284	0.9548	0.9027	0.8864	1.8672	1.7951	1.6372	1.5268	1.3967
0.055	1.8592	1.6471	1.3964	1.0983	0.9257	2.195	2.0672	1.8549	1.6273	1.4967

Panel B: Stock Return

	$\mu_{Q,t}$ at t=0					$\mu_{Q,t}$ at t^{*+}				
	$\bar{s}$=0.9	0.8	0.7	0.6	0.5	$\bar{s}$=0.9	0.8	0.7	0.6	0.5
τ_a=0.015	0.0127	0.0198	0.0216	0.0289	0.0317	0.0269	0.0384	0.0407	0.0468	0.0523
0.025	0.0846	0.0764	0.0637	0.0718	0.0824	0.1692	0.1437	0.1329	0.1568	0.1634
0.035	0.1974	0.1643	0.1786	0.1857	0.1908	0.3492	0.3218	0.2917	0.2643	0.2185
0.045	0.2861	0.2457	0.2093	0.1839	0.1768	0.6491	0.5637	0.4813	0.4072	0.3659
0.055	0.3942	0.2967	0.2348	0.2041	0.1936	0.9462	0.8027	0.7364	0.5619	0.3018

Panel C: Volatility of Market Value

	$\sigma_{Q,t}$ at t=0					$\sigma_{Q,t}$ at t^{*+}				
	$\bar{s}$=0.9	0.8	0.7	0.6	0.5	$\bar{s}$=0.9	0.8	0.7	0.6	0.5
τ_a=0.015	0.0633	0.0798	0.0854	0.1096	0.1342	0.1384	0.1167	0.0939	0.1167	0.1463
0.025	0.1386	0.1297	0.1204	0.1138	0.1253	0.1857	0.1764	0.1709	0.1628	0.1726
0.035	0.2761	0.2637	0.2549	0.2316	0.2478	0.3968	0.3725	0.3547	0.3103	0.3269
0.045	0.4837	0.3961	0.3624	0.3258	0.3309	0.6749	0.6218	0.5943	0.5702	0.6031
0.055	0.7392	0.6928	0.6735	0.6083	0.5437	1.083	0.9617	0.9328	0.8862	0.7629

Internet Appendix for "Pricing Voluntary Disclosure"

Shuting Hou, Rui Sun, and Hulai Zhang

IA.1 Proofs

Proof of Proposition 1. Since the evolution of stock issued by an arbitrary firm j given by $dS_t^j = (1 - s_t)S_t^j dr_t^j$. The book value of the stock issued by firm j at T can be expressed as

$$S_T^j = S_{t^*}^j e^{(1-s)((\mu+a-\frac{(1-s)(\sigma_r^2+\tau_\iota^2)}{2}-\frac{(1-s)\sigma_e^2}{2})(T-t^*)+\sigma_r(W_T-W_{t^*})+\int_{t^*}^T \iota_u dW_u+\sigma_e(W_T^j-W_{t^*}^j))}$$

where s is the optimal saving rate we derived in Proposition 2, $a = a_0$ without a disclosure change, and $a = a_1$ if the firm choose to disclose. Thus, the aggregate book value of stocks across all the firms is

$$\begin{aligned} S_T &= \int_0^1 S_T^j dj \\ &= e^{(1-s)((\mu+a-\frac{(1-s)(\sigma_r^2+\tau_\iota^2)}{2}-\frac{(1-s)\sigma_e^2}{2})(T-t^*)+\sigma_r(W_T-W_{t^*})+\int_{t^*}^T \iota_u dW_u)} \int_0^1 S_{t^*}^j e^{(1-s)\sigma_e(W_T^j-W_{t^*}^j)} dj \end{aligned} \tag{IA.1}$$

By the law of large numbers and the independence of the Brownian motion, the integral in equation IA.1 is equal to

$$\begin{aligned} \int_0^1 S_{t^*}^j e^{(1-s)\sigma_e(W_T^j-W_{t^*}^j)} dj &= E_{t^*}^j(S_{t^*}^j e^{(1-s)\sigma_e(W_T^j-W_t^j)})dj \\ &= E_{t^*}^j(S_{t^*}^j)E_{t^*}^j(e^{(1-s)\sigma_e(W_T^j-W_t^j)}) \\ &= S_{t^*} e^{\frac{(1-s)^2\sigma_e^2}{2}(T-t^*)} \end{aligned} \tag{IA.2}$$

Plugging IA.2 into IA.1, we can obtain the expression of S_T as claimed in Proposition 1

$$S_T = S_{t^*} e^{(1-s)((\mu+a-\frac{(1-s)(\sigma_r^2+\tau_\iota^2)}{2}-\frac{(1-s)\sigma_e^2}{2})(T-t^*)+\frac{(1-s)^2\sigma_e^2}{2}(T-t^*)+\sigma_r(W_T-W_{t^*})+\int_{t^*}^T \iota_u dW_u))}$$
$$= S_{t^*} e^{(1-s)((\mu+a-\frac{(1-s)(\sigma_r^2+\tau_\iota^2)}{2})(T-t^*)+\sigma_r(W_T-W_{t^*})+\int_{t^*}^T \iota_u dW_u))}$$

■

Proof of Proposition 1. Assume that the optimal path of consumption and portfolio choice after an arbitrary time t satisfy that $c_l = \bar{c}_l$ and $s_l = \bar{s}_l$ for $l \geq t$, what remains to show is that $c_l = \bar{c}_l$ and $s_l = \bar{s}_l$ are also true when $l < t$. For $l \geq t$, the evolution of the shareholder's asset holdings is

$$\begin{aligned} dA_l &= (1-\bar{s}_l)A_l((\mu_r + a_l)dl + (\sigma_r + \iota_l))dW_l) + A_l\bar{s}_l r_f dl - \bar{c}_l dl \\ dx_l &= (b_2\bar{c}_l - b_1 x_l)dl \end{aligned}$$

Combining these two equations, we can have

$$d(A_l - \frac{x_l}{r_f + b_1 - b_2}) = [(\mu + a_l - r_f)(1-\bar{s}_l) + r_f)A_l - \bar{c}_l - \frac{b_2\bar{c}_l - b_1 x_l}{r_f + b_1 - b_2}]dl$$
$$+ (\sigma_r + \iota_l)(1-\bar{s}_l)A_l dW_l$$

$$d(A_l - \frac{x_l}{r_f + b_1 - b_2}) = (A_l - \frac{x_l}{r_f + b_1 - b_2})(Z_2 dl + Z_1(\sigma_r + \iota_l)dW_l)$$

where Z_1, Z_2, and are expressions defined in Section 2. Therefore the shareholder's asset holdings can be expressed as

$$A_l - \frac{x_l}{r_f + b_1 - b_2} = (A_t - \frac{x_t}{r_f + b_1 - b_2})e^{(Z_2 - \frac{Z_1^2(\sigma_r^2+\tau_\iota^2)}{2})(l-t)+Z_1\sigma_r(W_l - W_t)+Z_1\int_t^l \iota_u dW_u}, l > t$$

Therefore the results in Proposition 1 are reached. ■

Proof of Proposition 2. For the term in the shareholder's utility function when $l \geq t^*$, it can be written in the following way using the expression of the shareholder's optimal asset holdings given by Proposition 1.

$$\begin{aligned} &E_t[e^{-\beta(l-t)}[\bar{c}_l - x_l]^{1-\phi}] \\ =&Z_{4t}^{1-\phi}[A_t - \frac{x_t}{r_f + b_1 - b_2}]^{1-\phi} e^{(l-t)(-\beta+(1-\phi)(Z_2 - \frac{(\sigma_r^2+\tau_\iota^2)Z_1^2}{2}) + \frac{(1-\phi)^2 Z_1^2(\sigma_r^2+\tau_\iota^2)}{2})} \end{aligned} \tag{IA.3}$$

Based on the expression of Z_{4t}, the term in big bracket of the exponential can be written as

$$\begin{aligned} &-\beta + (1-\phi)(Z_2 - \frac{(\sigma_r^2+\tau_\iota^2)Z_1^2}{2}) + \frac{(1-\phi)^2 Z_1^2(\sigma_r^2+\tau_\iota^2)}{2} \\ = &-\frac{1}{\phi}(-\beta + (1-\phi)r_f - \frac{(1-\phi)(\mu + a_t - r_f)^2}{2\phi(\sigma_r^2+\tau_\iota^2)}) = -\frac{Z_{4t}(r_f+b_1)}{r_f+b_1-b_2} \end{aligned}$$

Therefore equation IA.3 becomes

$$Z_{4t}^{1-\phi}[A_t - \frac{x_t}{r_f + b_1 - b_2}]^{1-\phi} e^{-\frac{(r_f+b_1)(l-t)Z_{4t}}{r_f+b_1-b_2}} \tag{IA.4}$$

Plug equation IA.3 into the lifetime utility of the shareholder, we can define the value function with state variables A_t, x_t.

$$H(A_t, x_t) = E_t \int_t^\infty e^{-\beta(u-t)} \frac{(\bar{c}_l - x_l)^{1-\phi}}{1-\phi} dl = (A_t - \frac{x_t}{r_f + b_1 - b_2})^{1-\phi} \frac{Z_{4t}^{-\phi}(r_f + b_1 - b_2)}{(r_f + b_1)(1-\phi)}$$

We define G_t in the following way

$$G_t = \int_0^t e^{\beta s} \frac{(c_l - x_l)^{1-\phi}}{1-\phi} dl + e^{-\beta t} H(A_t, x_t)$$

By Ito's formula, we have

$$dG_t = L_t dt + e^{-\beta t}(1 - s_t)(\sigma_r + \iota_t)A_t H_{A_t} dW_t$$

Where H_{A_t} is the derivative of $H(A_t, x_t)$ with respect to A_t, and L_t is given as

$$\begin{aligned} L_t = e^{-\beta t}\{\frac{(c_t - x_t)^{1-\phi}}{1-\phi} - \beta H(A_t, x_t) + [(\mu + a_t - r_f)(1 - s_t)A_t + r_f A_t - c_t]H_{A_t} \\ + \frac{(1 - s_t)^2(\sigma_r + \iota_t)^2}{2} A_t^2 H_{AA_t} + (b_2 c_t - b_1 x_t)H_{x_t}]\} \end{aligned}$$

where H_{AA_t} is the second order derivative of $H(A_t, x_t)$ with respect to A_t. Then, by taking derivative of L_t with respect to c_t and s_t, we can solve the optimal consumption and saving ratio

$$\bar{c}_t = x_t + Z_{4t}(A_t - \frac{x_t}{r_f + b_1 - b_2})$$

$$\bar{s}_t = 1 - Z_1[1 - \frac{x_t}{A_t(r_f + b_1 - b_2)}]$$

Taking logs on the expression of $\bar{c}_t - x_t$, we have

$$\ln(\bar{c}_t - x_t) = \ln(Z_{4t}) + \ln(A_t - \frac{x_t}{r_f + b_1 - b_2}) \tag{IA.5}$$

Combining equation IA.5 and the results of Proposition 1, the following equation is satisfied

$$\begin{aligned} &\ln(\bar{c}_t - x_t) \\ =&\ln(Z_{4t}) + \ln(A_0 - \frac{x_0}{r_f + b_1 - b_2}) + (Z_2 - \frac{Z_1^2(\sigma_r^2 + \tau_\iota^2)}{2})t + Z_1(\sigma_r W_t + \int_0^t \iota_u dW_u) \end{aligned}$$

Hence, we can have the following condition

$$\frac{d(\bar{c}_t - x_t)}{\bar{c}_t - x_t} = Z_2 dt + Z_1(\sigma_r + \iota_t) dW_t$$

Solving $\frac{d\bar{c}_t}{\bar{c}_t}$ based on the above equation, the evolution of the optimal consumption growth rate can be obtained

$$\begin{aligned}
\frac{d\bar{c}_t}{\bar{c}_t} &= \frac{dx_t + (\bar{c}_t - x_t)Z_2 dt + (\bar{c}_t - x_t)(Z_1(\sigma_r + \iota_t)dW_t)}{\bar{c}_t} \\
&= \frac{b_2\bar{c}_t - b_1 x_t + (\bar{c}_t - x_t)Z_2}{\bar{c}_t} dt + (1 - \frac{x_t}{\bar{c}_t})(Z_1(\sigma_r + \iota_t)dW_t) \\
&= [Z_2 + b_2 - \frac{(Z_2 + b_1)x_t}{\bar{c}_t}]dt + (1 - \frac{x_t}{\bar{c}_t})(Z_1(\sigma_r + \iota_t)dW_t)
\end{aligned}$$

which is what we claimed in Proposition 2.

As for the $h_t = \frac{x_t}{c_t}$, deriving h_t with respect to time gives us

$$dh_t = [b_2 - (Z_2 + b_1 - Z_1^2(\sigma_r^2 + \tau_\iota^2)h_t - Z_1^2(\sigma_r^2 + \tau_\iota^2)h_t^2](1 - h_t)dt - h_t(1 - h_t)Z_1(\sigma_r^2 + \tau_\iota^2)dW_t$$

Let $Y_t = \frac{h_t}{1 - h_t}$, then the equation above becomes

$$dY_t = [b_2 - (Z_2 + b_1 - b_2 - (Z_1^2(\sigma_r^2 + \tau_\iota^2)Y_t]dt - Y_t(Z_1(\sigma_r + \iota_t)dW_t) \tag{IA.6}$$

Deriving equation IA.6 with respect to time, we have

$$\frac{\partial(Y_t^2 Z_1^2(\sigma_r^2 + \tau_\iota^2)P(Y_t))}{2\partial Y_t} - [b_2 - (Z_2 + b_1 - b_2 - Z_1^2(\sigma_r^2 + \tau_\iota^2)Y_t]P(Y_t) = 0 \tag{IA.7}$$

where $P(Y_t)$ satisfies $\frac{dP(Y_t)}{dY_t} = 0$. Thus the equation IA.7 can be simplified to

$$b_2 - (Z_2 + b_1 - b_2)Y_t = 0$$

By the properties of the Pearson equation, we have $P(Y_t) = \frac{h_t}{1-h_t}$. Hence, $P(h_t)$ can be written in the following way

$$P(h_t) = P(Y_t)\frac{dY_t}{dh_t} = P(Y_t)\frac{1}{(1-h_t)^2} = P(\frac{h_t}{1-h_t})\frac{1}{(1-h_t)^2}$$

Since $\frac{dP(Y_t)}{dY_t} = 0$, $\frac{dP(h_t)}{dh_t} = 0$. Therefore, the following equation is true

$$Z_1^2(\sigma_r^2 + \tau_\iota^2)\bar{h}_t^2 - (Z_2 + b_1)\bar{h}_t + b_2 = 0$$

By solving the above equation, we have the expression of the optimal ratio of habit formation to consumption

$$\bar{h}_t = \frac{Z_2 + b_1 - \sqrt{(Z_2 + b_1)^2 - 4Z_1^2(\sigma_r^2 + \tau_\iota^2)b_2}}{2Z_1^2(\sigma_r^2 + \tau_\iota^2)^2} \tag{IA.8}$$

which is what we claimed in Proposition 2. Therefore, $\frac{x_t}{c_t}$ is a constant, when the consumer is on the optimal path of consumption and saving. Combining the optimal consumption path given by Proposition 2 and IA.8, $\frac{x_t}{A_t}$ is also an constant in the optimal case. Furthermore, with a constant optimal $\frac{x_t}{A_t}$, the optimal saving rate $\bar{s}_t$ is also a constant. ■

Proof of Proposition 3. The expected utility of the shareholder conditional on the information set of the firm, which includes the cost of disclosure change C. Also, for the effectiveness of the disclosure a_t, it follows $N(\hat{a}_{t^*}, \hat{\tau}_{a_{t^*}})$ without a disclosure change, and $N(0, \tau_a^2)$ when disclosure happens. Hence the firm compares $E_{t^*}[V(A_{t^*}, x_{t^*})|Old]$ with $E_{t^*}[CV(A_{t^*}, x_{t^*})|New]\}$ to decide whether to disclose. Based on the evolution of A_t given by Equation (5) and the independence of the Brownian motion W_t, $E_{t^*}[V(A_{t^*}, x_{t^*})|Old]$ and

$E_{t^*}[CV(A_{t^*}, x_{t^*})|New]\}$ can be written as

$$
\begin{aligned}
&E_{t^*}[CV(A_{t^*}, x_{t^*})|New] \\
=&E_{t^*}[\int_{t^*}^{T} \frac{C(c_t - x_t)^{1-\phi}}{1-\phi} dt|New] \\
=&\frac{(c_{t^*} - x_{t^*})^{1-\phi}}{1-\phi} \frac{1}{Z_2^D + \frac{Z_1^D(\sigma_r+\tau_\iota)}{2}} e^{c+Z_2^D(T-t^*)+\frac{Z_1^D(\sigma_r+\tau_\iota)(T-t^*)}{2}}
\end{aligned}
\tag{IA.9}
$$

$$
\begin{aligned}
&E_{t^*}[V(A_{t^*}, x_{t^*})|Old] \\
=&E_{t^*}[\int_{t^*}^{T} \frac{(c_t - x_t)^{1-\phi}}{1-\phi} dt|Old] \\
=&\frac{(c_{t^*} - x_{t^*})^{1-\phi}}{1-\phi} \frac{1}{Z_2^{ND} + \frac{Z_1^{ND}(\sigma_r+\tau_\iota)}{2}} e^{Z_2^{ND}(T-t^*)+\frac{Z_1^{ND}(\sigma_r+\tau_\iota)(T-t^*)}{2}}
\end{aligned}
\tag{IA.10}
$$

where Z_1^{ND},Z_1^D, Z_2^{ND}, and Z_2^D are defined as

$$
Z_1^D = \frac{\mu + a_1 - r_f}{\phi(\sigma_r + \tau_\iota^2)}, Z_1^{ND} = \frac{\mu + \hat{a}_{t^*} - r_f}{\phi(\sigma_r + \tau_\iota^2)}
$$

$$
Z_2^D = \frac{r_f - \beta}{\phi} + \frac{(\mu + a_1 - r_f)^2(1+\phi)}{2\phi^2(\sigma_r + \tau_\iota^2)}, Z_2^{ND} = \frac{r_f - \beta}{\phi} + \frac{(\mu + \hat{a}_{t^*} - r_f)^2(1+\phi)}{2\phi^2(\sigma_r + \tau_\iota^2)}
$$

When $a_{t^*} = a_0$, the expectation operator $E_{t^*}^{ND}$ corresponds to the belief $N(\hat{a}_{t^*}, \hat{\tau}_{a_{t^*}})$ of a_{t^*}, and to the belief $N(0, \tau_a)$ of a_{t^*} when $a_{t^*} = a_1$. Then the threshold of disclosure change $a(\underline{C})$ and the corresponding probability of disclosure change at t^*, $q(\hat{a}_{t^*})$ can be solved numerically by equating equation IA.9 and equation IA.10, which is.

$$
E_{t^*}[V(A_{t^*}, x_{t^*})|Old] = E_{t^*}[CV(A_{t^*}, x_{t^*})|New]
$$

■

Proof of Proposition 4. Suppose we are in a complete market, the price of an asset Ω can be defined as

$$Pr(\Omega) = \sum_{st=0}^{st^*} P(st)\Omega(st)$$

where $st \in \{0, 1, 2, 3, ..., st^*\}$ denotes the state of the market, $\Omega(st)$ stands for the return of this asset in state st, and $P(st)$ is the state price density is in state st. In our analysis above, the book value of a stock can be regarded as the return $\Omega(st)$, and the market value of a stock in state st can be expressed as $Pr(st)$. Therefore, for a time t during the interval $[0, T]$, the market value of the stock of firm j satisfies the following equation

$$Q_t^j = E_t[\frac{P_T}{P_t} S_T^j]$$

We assume that a firm only values the book value of its stock. Also, the firm shares the same utility function of the shareholder since the shareholder owns it. Thus, the maximization problem of firm j can be formulated as

$$\max_{\{S_{st}^j\}_{st \in \{e,0,1,2,3,...st^*\}}} U(S_e^j) + \sum_{st=0}^{st^*} \beta^f \pi(st) U(S_{st}^j)$$

$$st.S_e^j + \sum_{st=0}^{st^*} P(st) S_{st}^j = \Theta_e^j + \sum_{st=0}^{st^*} P(st)\Theta_{st}^j$$

where $\pi(st)$ is the probability that the market is state st, β^f is the discount factor of a firm, and Θ_{st}^j denotes the total assets owned by the firm j in state st. Solving the maximization problem, the first order conditions are

$$U^{'}(S_e^j) = \lambda$$

$$\beta^f \pi(st) U^{'}(S_{et}^j) = \lambda P(st), st \in \{0, 1, 2, 3..., st^*\}$$

where λ is the Lagrangian multiplier associated with the budget constraint of the firm. Therefore, the state price density, $P(st)$ can be written as

$$P(st) = \frac{\beta^f \pi(st) U^{'}(S_{et}^j)}{\lambda}$$

Combined with the expression of the utility function and the optimal saving rate $\bar{s}$, the state price density at time t is given as

$$P_t = \frac{E_t(S_T^{-\phi})}{\lambda^{'}}$$

where $\lambda^{'} = \frac{\lambda}{\beta^f}$. And this is the expression we claim for the state price density at time t. Then at time t^*, the state price density is

$$P_{t^{*+}} = \frac{S_{t^{*+}}^{-\phi} E_t\left(e^{-\phi(1-\bar{s})((\mu+a-\frac{(1-\bar{s})(\sigma_r^2+\tau_\iota^2)}{2})(T-t^*)+\sigma_r(W_T-W_{t^*})+\int_{t^*}^T \iota_u dW_u)}\right)}{\lambda^{'}} \tag{IA.11}$$

Using the expression of S_t given by Proposition 1, the right hand side of equation IA.11 can be specified in the following way based on whether there is a disclosure change at t^*

$$P_{t^{*+}} = \begin{cases} P_{t^{*+}}^D = \dfrac{S_{t^{*+}}^{-\phi} e^{-\phi(1-\bar{s})(\mu-(1+\phi)(1-\bar{s})(\sigma_r^2+\tau_\iota^2))(T-t^*)+\frac{\phi^2(1-\bar{s})^2\tau_a^2}{2}(T-t^*)^2}}{\lambda^{'}} & \text{Disclose} \\ P_{t^{*+}}^{ND} = \dfrac{S_{t^{*+}}^{-\phi} e^{-\phi(1-\bar{s})(\mu+\hat{a}_{t^*}-(1+\phi)(1-\bar{s})(\sigma_r^2+\tau_\iota^2))(T-t^*)+\frac{\phi^2(1-\bar{s})^2\hat{\tau}_{a_{t^*}}^2}{2}(T-t^*)^2}}{\lambda^{'}} & \text{Not Disclose} \end{cases}$$

At t^*, the state price density can be given as

$$P_{t^*} = E_{t^*}(P_{t^{*+}}) = q_{t^*} P_{t^{*+}}^D + (1-q_{t^*}) P_{t^{*+}}^{ND}$$

where $q_{t^*} = q(\hat{a}_{t^*})$ is the probability disclosure happens based on the belief of consumers.

The market value of the stock issued by firm j is given by $Q_t^j = E_t(\frac{P_T}{P_t} S_T^j)$. After t^*, the market vale can be given as

$$\begin{aligned} Q_{t^*}^j &= \lambda'^{-1} P_{t^*}^{-1} E_{t^*}(S_T^{-\phi} S_T^j) = \lambda'^{-1} P_{t^*}^{-1} E_{t^*}(E_{t^{*+}}(S_T^{-\phi} S_T^j)) \\ &= (q_{t^*} P_{t^{*+}}^D + (1-q_{t^*}) P_{t^{*+}}^{ND})^{-1} \lambda'^{-1} (q_{t^*} E_{t^{*+}}(S_T^{-\phi} S_T^j | ND) + (1-q_{t^*}) E_{t^{*+}}(S_T^{-\phi} S_T^j | Old)) \end{aligned}$$

where $E_{t^{*+}}(S_T^{-\phi} S_T^j | New)$ and $E_{t^{*+}}(S_T^{-\phi} S_T^j | Old)$ can be obtained using the expression of S_T and S_T^j derived in Proposition 1.

$$E_{t^{*+}}(S_T^{-\phi} S_T^j | New) = S_{t^{*+}}^{-\phi} S_{t^{*+}}^j e^{(1-\bar{s})(1-\phi)(\mu - \frac{\phi(1-\bar{s})}{2}(\sigma_r^2 + \tau_a^2))(T-t^*) + \frac{(1-\phi)^2(1-\bar{s})^2(T-t^*)^2)\tau_a^2}{2}}$$

$$E_{t^{*+}}(S_T^{-\phi} S_T^j | Old) = S_{t^{*+}}^{-\phi} S_{t^{*+}}^j e^{(1-\bar{s})(1-\phi)(\mu + \hat{a}_{t^*} - \frac{\phi(1-\bar{s})}{2}(\sigma_r^2 + \hat{\tau}_{a_{t^*}}^2))(T-t^*) + \frac{(1-\phi)^2(1-\bar{s})^2(T-t^*)^2 \hat{\tau}_{a_{t^*}}^2}{2}}$$

Therefore, the market value just after the time of disclosure change t^* is

$$Q_{t^{*+}}^j = \begin{cases} Q_{t^{*+}}^{j,New} & = (\lambda' P_{t^{*+}}^D)^{-1} E_{t^{*+}}(S_T^{-\phi} S_T^j | New) \\ & = S_{t^{*+}}^j e^{(1-\bar{s})(\mu - \phi(1-\bar{s})(\sigma_r^2 + \tau_\iota^2))(T-t^*) + \frac{(1-2\phi)(1-\bar{s})^2 \tau_a^2 (T-t^*)^2}{2}}, \text{D} \\ Q_{t^{*+}}^{j,Old} & = (\lambda' P_{t^{*+}}^{ND})^{-1} E_{t^{*+}}(S_T^{-\phi} S_T^j | Old) \\ & = S_{t^{*+}}^j e^{(1-\bar{s})(\mu + \hat{a}_{t^*} - \phi(1-\bar{s})(\sigma_r^2 + \tau_\iota^2))(T-t^*) + \frac{(1-2\phi)(1-\bar{s})^2 \hat{\tau}_{a_{t^*}}^2 (T-t^*)^2}{2}}, \text{ND} \end{cases}$$

We can define the weight of $Q_{t^{*+}}^D$ as

$$\Theta = \frac{q_{t^*} P_{t^{*+}}^D}{q_{t^*} P_{t^{*+}}^D + (1-q_{t^*}) P_{t^{*+}}^{ND}}$$

From results in the Proposition 3, during the disclosure interval, the state price density at time t can be expressed as $P_t = \frac{E_t(S_T^{-\phi})}{\lambda'}$. Its value right after the time of disclosure change t^* is given by

$$P_{t^*} = E_t(P_{t^{*+}}) = \int E_t(P_{t^{*+}} | \ln(C)) f(\ln(C)) d\ln(C)$$

where $f(\ln(C))$ is the probability density function of the normal distribution with mean $-\frac{\tau_c^2}{2}$, variance τ_c^2. Moreover, according to whether the firm disclose or not, the conditional expectation $E_t(P_{t^{*+}} | \ln(C))$ can be decomposed into

$$E_t(P_{t^{*+}} | \ln(C)) = q(\hat{a}_{t^*}) E_t(P_{t^{*+}} | \hat{a}_{t^*} < \underline{a}(C), \ln(C)) + (1 - q(\hat{a}_{t^*})) E_t(P_{t^{*+}} | \hat{a}_{t^*} > \underline{a}(C), \ln(C)) \quad \text{(IA.12)}$$

Where $\underline{a}(C)$ and $q(\hat{a}_{t^*})$ are determined by the following condition based on the distribution of $\hat{a}_t$

$$q(\hat{a}_{t^*}) = F_N(\hat{a}_t, \hat{\tau}_{a_t}^2 - \hat{\tau}_{a_{t^*}}^2; \underline{a}(C))$$

When disclosure happens, the price density process P_{t^*+} is given by the following equation based on equation IA.11.

$$P_{t^*+} = \frac{S_{t^*+}^{-\phi} e^{-\phi(1-\bar{s})(\mu - (1+\phi)(1-\bar{s})(\sigma_r^2 + \tau_\iota^2))(T-t^*) + \frac{\phi^2(1-\bar{s})^2 \tau_a^2}{2}(T-t^*)^2}}{\lambda'}$$

A_{t^*+} can be denoted as A_{t^*} because A_t is a continuous process. Therefore, the first conditional expectation in equation IA.12 can be calculated as

$$E_t(P_{t^*+}|\hat{a}_{t^*} < \underline{a}(C), \ln(C))$$
$$= \frac{e^{-\phi(1-\bar{s})(\mu-(1+\phi)(1-\bar{s})(\sigma_r^2+\tau_\iota^2))(T-t^*)+\frac{\phi^2(1-\bar{s})^2\tau_a^2}{2}(T-t^*)^2}}{\lambda'} E_t(S_{t^*}^{-\phi}|\hat{a}_{t^*} < \underline{a}(C), \ln(C))$$

The process of the aggregate asset holdings can be expressed as

$$\frac{S_{t^*}}{S_t} = e^{(1-s_t)(\mu(t^*-t)+\int_t^{t^*}\hat{a}_l dl - \frac{\sigma_r^2+\tau_\iota^2}{2}(t^*-t)+\sigma_r(W_{t^*}-W_t)+\int_t^{t^*}\iota_l dWl)}$$

By Ito's lemma, we can have the joint process for $ln(S_t)$ and $\hat{a}_t$

$$dln(S_t) = (1-s_t)((\mu + \hat{a}_t - \frac{1}{2}(\sigma_r^2 + \tau_\iota^2))dt + (\sigma_r + \iota_t)dW_t)$$

$$d\hat{a}_t = \frac{\hat{\tau}_t^2}{\sigma_r + \iota_t} dW_t$$

By integration and with $s_t = \bar{s}$ in the optimal case, $ln(S_{t^*})$ and $\hat{\tau}_{t^*}$ can be written as

$$ln(S_{t^*}) = ln(S_t) + (1-\bar{s})(\mu(t^*-t) + \int_t^{t^*}(\hat{a}_l - \frac{1}{2}(\sigma_r^2+\tau_\iota^2))dl + \int_t^{t^*}(\sigma_r+\iota_l)dW_l)$$

$$\hat{a}_{t^*} = a_t + \int_t^{t^*} \frac{\hat{\tau}_l^2}{\sigma_r + \iota_l} dW_l$$

The expectation of $ln(S_{t^*})$ conditional on shareholder's information set at time t can be given as

$$E_t(ln(S_{t^*})) = ln(S_t) + (1-\bar{s})(\mu + \hat{a}_t - \frac{1}{2}(\hat{a}_t^2 + \tau_\iota^2))(t^*-t)$$

And the corresponding conditional variance is

$$Var(ln(S_{t^*})) = Var((1-\bar{s})(\int_t^{t^*}(\mu + \hat{a}_l - \frac{1}{2}(\hat{a}_l^2 + \tau_\iota^2))dl + \int_t^{t^*}(\sigma_r + \iota_l)dW_l)$$

When $t < t^*$, $\hat{a}_t \sim N(\hat{a}_t, \hat{\tau}_t^2)$. The variance can be written as

$$Var(ln(S_{t^*})) = (1-\bar{s})^2((t^*-t)^2\hat{\tau}_t^2 + (\sigma_r^2 + \tau_\iota^2)(t^*-t))$$

The conditional covariance between $ln(S_{t^*})$ and $\hat{a}_{t^*}$ is defined by

$$Cov(ln(S_{t^*}), \hat{a}_{t^*}) = E_t[\hat{a}_{t^*}ln(S_{t^*})] - E_t[ln(S_{t^*})]E_t[\hat{a}_{t^*}]$$

By Ito's Lemma, $d(\hat{a}_t ln(S_t))$ is given by

$$\begin{aligned} d(\hat{a}_t ln(S_t)) &= ln(S_t)d\hat{a}_t + \hat{a}_t dln(S_t) + d\hat{a}_t dln(S_t) \\ = (1-\bar{s})(\mu + \hat{a}_t - \frac{1}{2}(\sigma_r^2 + \tau_\iota^2)\hat{a}_t + \hat{\tau}_t^2)dt &+ ((1-\bar{s})(\sigma_r + \iota_t)\hat{a}_t + \frac{\hat{\tau}_t^2 ln(S_t)}{\sigma_r + \iota_t})dW_t \end{aligned} \tag{IA.13}$$

Integral both sides of equation IA.13 from t to t^*

$$\begin{aligned} \hat{a}_{t^*}ln(S_{t^*}) = \hat{a}_t ln(S_t) &+ \int_t^{t^*}(1-\bar{s})(\mu + \hat{a}_l - \frac{1}{2}(\sigma_r^2 + \tau_\iota^2)\hat{a}_l + \hat{\tau}_t^2)dl \\ &+ \int_t^{t^*}((1-\bar{s})(\sigma_r + \iota_l)\hat{a}_l + \frac{\hat{\tau}_l^2 ln(A_l)}{\sigma_r + \iota_l})dW_l \end{aligned}$$

Therefore the expectation of $\hat{a}_{t^*}ln(S_{t^*})$ conditional on information set at t is

$$E_t(\hat{a}_{t^*}ln(S_{t^*})) = \hat{a}_t ln(S_t) + (1-\bar{s})(\mu + \hat{a}_t^2 - \frac{1}{2}(\sigma_r^2 + \tau_\iota^2)\hat{a}_t + \hat{\tau}_t^2)(t^*-t)$$

where the second moment of $\hat{a}_t$ that $E_t(\hat{a}_l) = \hat{a}_t^2 + (\hat{\tau}_l^2 - \hat{\tau}_t^2)$ is used to obtain the evolution of $ln(S_t)$. Therefore, the conditional covariance between $ln(A_{t^*})$ and $\hat{a}_{t^*}$ can be calculated as

$$
\begin{aligned}
Cov(ln(S_{t^*}), \hat{a}_{t^*}) &= E_t[\hat{a}_{t^*} ln(S_{t^*})] - E_t[ln(S_{t^*})]E_t[\hat{a}_{t^*}] \\
= \hat{a}_t ln(S_t) &+ (1-\bar{s})(\mu + \hat{a}_t - \frac{1}{2}(\sigma_r^2 + \tau_\iota^2)\hat{a}_t + \hat{\tau}_t^2)(t^* - t) \\
-(ln(S_t) &+ (t^* - t)(1-\bar{s})(\mu + \hat{a}_t - \frac{1}{2}(\sigma_r^2 + \tau_\iota^2)))\hat{a}_t \\
&= (1-\bar{s})\hat{\tau}_t^2(t^* - t)
\end{aligned} \tag{IA.14}
$$

Since both $ln(S_{t^*})$ and $\hat{a}_{t^*}$ follow normal distributions, the conditional expectation $E_t(S_{t^*}^{-\phi}|\hat{a}_{t^*} < \underline{a}(C), \ln(C))$ can be calculated based on the moment conditions of $ln(S_{t^*})$ and $\hat{a}_{t^*}$ we derived above

$$
\begin{aligned}
E_t(S_{t^*}^{-\phi}|\hat{a}_{t^*} < \underline{a}(C), \ln(C)) &= \int_{-\infty}^{\underline{a}(C)} E_t(S_{t^*}|\hat{a}_{t^*} = u) f_a(u|\hat{a}_{t^*} < \underline{a}(C), \ln(C))du \\
= \int_{-\infty}^{\underline{a}(C)} e^{-\phi(E_t(ln(S_{t^*}) + \frac{Cov(ln(S_{t^*}, \hat{a}_{t^*}))}{Var(\hat{a}_{t^*})}(u - \hat{a}_t) + \frac{\phi^2}{2}(Var(ln(S_{t^*}) - \frac{Cov(ln(S_{t^*}), \hat{a}_{t^*})}{Var(\hat{a}_{t^*})})^2} & f_a(u|\hat{a}_{t^*} < \underline{a}(C), \ln(C))du
\end{aligned} \tag{IA.15}
$$

where the probability density function of $\hat{a}_{t^*}$ is given by

$$
f_a(u|\hat{a}_{t^*} < \underline{a}(C), \ln(C)) = \frac{f_N(\hat{a}_t, Var(\hat{a}_t); u)}{F_N(\hat{a}_t, \hat{\tau}_t^2 - \hat{\tau}_{t^*}^2; \underline{a}(C))} \tag{IA.16}
$$

where $f_N(x, y; z)$ is the probability density of normal distribution with mean x, variance y at point z. Plug equation IA.15 into equation IA.16

$$
E_t(S_{t^*}^{-\phi}|\hat{a}_{t^*} < \underline{a}(C), \ln(C)) = \frac{F_N(\hat{a}_t - \phi Cov(ln(S_{t^*}), \hat{a}_{t^*}), \hat{\tau}_t^2 - \hat{\tau}_{t^*}^2; \underline{a}(C))}{F_N(\hat{a}_t, \hat{\tau}_t^2 - \hat{\tau}_{t^*}^2; \underline{a}(C))} e^{-\phi(E_t(ln(S_{t^*})) + \frac{\phi^2}{2}Var(ln(S_{t^*})))} \tag{IA.17}
$$

Combing the expression of $Cov(ln(S_{t^*}), \hat{a}_{t^*})$, $E_t(ln(S_{t^*}))$, and $Var(ln(S_{t^*}))$, equation IA.17 can be transformed into

$$E_t(S_{t^*}^{-\phi}|\hat{a}_{t^*} < \underline{a}(C), \ln(C)) = \quad S_t^{-\phi} e^{-\phi(1-\bar{s})(\mu+\hat{a}_t)(t^*-t)+\frac{\phi^2}{2}(1-\bar{s})^2(t^*-t)^2\hat{\tau}_{a_t}^2+\frac{\phi^2}{2}(1-\bar{s})^2(\sigma_r^2+\tau_\iota^2)(t^*-t)} \frac{F_N(\hat{a}_t-\phi Cov(ln(S_{t^*}),\hat{a}_{t^*}),\hat{\tau}_{a_t}^2-\hat{\tau}_{a_{t^*}}^2;\underline{a}(C))}{F_N(\hat{a}_t,\hat{\tau}_{a_t}^2-\hat{\tau}_{a_{t^*}}^2;\underline{a}(C))}$$

When there is a disclosure change, the price density just after the changing date can be defined in the following equation as we derived before

$$P_{t^*+} = \frac{S_{t^*+}^{-\phi} e^{-\phi(1-\bar{s})(\mu-(1+\phi)(1-\bar{s})(\sigma_r^2+\tau_\iota^2))(T-t^*)+\frac{\phi^2(1-\bar{s})^2\tau_a^2}{2}(T-t^*)^2}}{\lambda'}$$

Therefore, the conditional expectation of the state price density when disclosure happens, $E_t(P_{t^*+}|\hat{a}_{t^*} < \underline{a}(C), \ln(C))$ can be written as

$$\begin{aligned}
&E_t(P_{t^*+}|\hat{a}_{t^*} < \underline{a}(C), \ln(C)) \\
=&\frac{e^{-\phi(1-\bar{s})(\mu-(1+\phi)(1-\bar{s})(\sigma_r^2+\tau_\iota^2))(T-t^*)+\frac{\phi^2(1-\bar{s})^2\tau_a^2}{2}(T-t^*)^2}}{\lambda'} E_t(S_{t^*}^{-\phi}|\hat{a}_{t^*} < \underline{a}(C), \ln(C)) \\
=&\frac{e^{-\phi(1-\bar{s})(\mu-(1+\phi)(1-\bar{s})(\sigma_r^2+\tau_\iota^2))(T-t^*)+\frac{\phi^2(1-\bar{s})^2\tau_a^2}{2}(T-t^*)^2-\phi(1-\bar{s})(\mu+\hat{a}_t)(t^*-t)+\frac{\phi^2}{2}(1-\bar{s})^2(t^*-t)^2\hat{\tau}_{a_t}^2+\frac{\phi^2}{2}(1-\bar{s})^2(\sigma_r^2+\tau_\iota^2)(t^*-t)}}{\lambda'} S_t^{-\phi} \\
&\times \frac{F_N(\hat{a}_t - \phi Cov(ln(S_{t^*}), \hat{a}_{t^*}), \hat{\tau}_{a_t}^2 - \hat{\tau}_{a_{t^*}}^2; \underline{a}(C))}{F_N(\hat{a}_t, \hat{\tau}_{a_t}^2 - \hat{\tau}_{a_{t^*}}^2; \underline{a}(C))}
\end{aligned}$$

When there is no disclosure change, the state price density just after t^* is presented by the following equation

$$P_{t^*+} = \frac{S_{t^*+}^{-\phi} e^{-\phi(1-\bar{s})(\mu+\hat{a}_{t^*}-(1+\phi)(1-\bar{s})(\sigma_r^2+\tau_\iota^2))(T-t^*)+\frac{\phi^2(1-\bar{s})^2\hat{\tau}_{a_{t^*}}^2}{2}(T-t^*)^2}}{\lambda'}$$

Then the conditional expectation of $E_t(P_{t^*+}|\hat{a}_{t^*} \geq \underline{a}(C), \ln(C))$ is given by

$$E_t(P_{t^*+}|\hat{a}_{t^*} \geq \underline{a}(C), \ln(C)) = e^{-\phi(1-\bar{s})(\mu-(1+\phi)(1-\bar{s})(\sigma_r^2+\tau_\iota^2))(T-t^*)+\frac{\phi^2(1-\bar{s})^2\hat{\tau}^2_{a_{t^*}}}{2}(T-t^*)^2} \times E_t(S_{t^*+}^{-\phi} e^{-\phi(1-\bar{s})\hat{a}_{t^*}(T-t^*)}\lambda'^{-1}|\hat{a}_{t^*} \geq \underline{a}(C), \ln(C)) \tag{IA.18}$$

To simplify the notation, we define Ψ_t as

$$\Psi_t = (1-\bar{s})\hat{a}_t(T-t) + ln(S_t)$$

The variance of Ψ_{t^*} can be computed as

$$\begin{aligned}
&Var(\Psi_{t^*}) \\
=&(T-t^*)^2(1-\bar{s})^2 Var(\hat{a}_{t^*}) + Var(ln(S_{t^*})) + 2(T-t^*)(1-\bar{s})Cov(ln(S_{t^*}), \hat{a}_{t^*}) \\
=&(1-\bar{s})^2(T-t^*)^2(\hat{\tau}^2_{a_t} - \hat{\tau}^2_{a_{t^*}}) + (1-\bar{s})^2(t^*-t)^2\hat{\tau}^2_{a_t} + (1-\bar{s})^2(t^*-t)(\sigma_r^2+\tau_\iota^2) \\
&+ 2(1-\bar{s})^2(t^*-t)\hat{\tau}^2_{a_t}(T-t^*) \\
=&(1-\bar{s})^2\hat{\tau}^2_{a_t}((T-t^*)^2 + (t^*-t)^2 + 2(T-t^2)(t^*-t)) - (1-\bar{s})^2(T-t^*)^2\hat{\tau}^2_{a_{t^*}} \\
&+ (1-\bar{s})^2(t^*-t)(\sigma_r^2+\tau_\iota^2) \\
=&(1-\bar{s})^2(\hat{\tau}^2_{a_t}(T-t)^2 - (T-t^*)^2\hat{\tau}^2_{t^*} + (\sigma_r^2+\tau_\iota^2)(t^*-t))
\end{aligned} \tag{IA.19}$$

where $Cov(\hat{a}^2_{t^*}, ln(S_{t^*}))$ is given by equation IA.14.

$$Cov(\hat{a}^2_{t^*}, ln(S_{t^*})) = (1-\bar{s})\hat{\tau}^2_{a_t}(t^*-t)$$

In the calculation of equation IA.19, we use the fact that $\hat{a}_{t^*}$ conditional on information set at t follows a normal distribution given by

$$N(\hat{a}_t, \hat{\tau}^2_t - \hat{\tau}^2_{t^*})$$

Moreover, the covariance between Ψ_{t^*} and $\hat{a}_{t^*}$ is

$$
\begin{aligned}
&Cov(\Psi_{t^*}, \hat{a}_{t^*}) = (1-\bar{s})(T-t^*)Var(\hat{a}_{t^*}) + Cov(ln(S_{t^*}), \hat{a}_{t^*}) \\
=&(1-\bar{s})(T-t^*)(\hat{\tau}_{a_t}^2 - \hat{\tau}_{a_{t^*}}^2) + (1-\bar{s})\hat{\tau}_t^2(t^*-t) \\
=&(1-\bar{s})((T-t)\hat{\tau}_{a_t}^2 - (T-t^*)\hat{\tau}_{a_{t^*}}^2)
\end{aligned}
$$

Based on the calculation above, the distribution of Ψ_{t^*} conditional on information set at t can be represented by the following normal distribution

$$
\Psi_{t^*} \sim N(E_t(\Psi_{t^*}) + \frac{Cov(\Psi_{t^*}, \hat{a}_{t^*})}{Var(\hat{a}_{t^*})}(\hat{a}_{t^*} - \hat{a}_t), Var(\Psi_{t^*}) - \frac{Cov(\Psi_{t^*}, \hat{a}_{t^*})^2}{Var(\hat{a}_{t^*})}) \tag{IA.20}
$$

Based on the distribution defined by equation IA.20, the conditional expectation in equation IA.18 can be transformed into

$$
\begin{aligned}
&E_t(S_{t^*}^{-\phi} e^{-\phi(1-\bar{s})\hat{a}_{t^*}(T-t^*)} | \hat{a}_{t^*} > \underline{a}(C), \ln(C)) = E_t(e^{-\phi\Psi_t} || \hat{a}_{t^*} > \underline{a}(C), \ln(C)) \\
=&\int_{\underline{a}(C)}^{\infty} e^{-\phi(E_t(\Psi_{t^*}) + \frac{Cov(\Psi_{t^*}, \hat{a}_{t^*})}{Var(\Psi_{t^*})}(u-\hat{a}_t)) + \frac{\phi^2}{2}(Var(\Psi_{t^*}) - \frac{Cov(\Psi_{t^*}, \hat{a}_{t^*})^2}{Var(\hat{a}_{t^*})})} f_a(u|\underline{a}(C), \ln(C))du
\end{aligned} \tag{IA.21}
$$

where $f_a(u|\underline{a}(C), \ln(C))$ is the conditional density function of a_{t^*}

$$
f_a(u|\underline{a}(C), \ln(C)) = \frac{f_N(\hat{a}_t, Var(\hat{a}_{t^*}); u)}{1 - F_N(\hat{a}_t, \hat{\tau}_{a_t}^2 - \hat{\tau}_{a_{t^*}}^2; \underline{a}(C))}
$$

Furthermore, after combining terms, equation IA.21 can be transformed into

$$
\begin{aligned}
&E_t(e^{-\phi\Psi_t} | \hat{a}_{t^*} > \underline{a}(C), \ln(C)) \\
=&\frac{1 - F_N(\hat{a}_t - \phi Cov(\Psi_{t^*}, \hat{a}_{t^*}), \hat{\tau}_t - \hat{\tau}_{t^*}^2; \underline{a}(C))}{1 - F_N(\hat{a}_t, \hat{\tau}_t^2 - \hat{\tau}_{t^*}^2; \underline{a}(C))} e^{-\phi E(\Psi_{t^*}) + \frac{\phi^2}{2} Var(\Psi_{t^*})}
\end{aligned} \tag{IA.22}
$$

Using equation IA.22 to substitute the conditional expectation in equation IA.18 can give us

$$E_t(P_{t^*+}|\hat{a}_{t^*} > \underline{a}(C), \ln(C)) = e^{-\phi(1-\bar{s})(\mu-(1+\phi)(1-\bar{s})(\sigma_r^2+\tau_\iota^2))(T-t^*)+\frac{\phi^2(1-\bar{s})^2\hat{\tau}^2_{a_{t^*}}}{2}(T-t^*)^2}$$
$$\times \frac{1-F_N(\hat{a}_t-\phi Cov(\Psi_{t^*}, \hat{a}_{t^*}), \hat{\tau}_t-\hat{\tau}^2_{t^*}; \underline{a}(C))}{1-F_N(\hat{a}_t, \hat{\tau}_t^2-\hat{\tau}^2_{t^*}; \underline{a}(C))} e^{-\phi E(\Psi_{t^*})+\frac{\phi^2}{2}Var(\Psi_{t^*})}$$

Combined with the expression of $E_t(P_{t^*+}|\hat{a}_{t^*} \leq \underline{a}(C), \ln(C))$, $E_t(P_{t^*+}|\ln(C))$ can be given as

$$\begin{aligned}
E_t(P_{t^*+}|\ln(C)) =& q(\hat{a}_{t^*})E_t(P_{t^*+}|\hat{a}_{t^*} \leq \underline{a}(C), \ln(C)) + (1-q(\hat{a}_{t^*}))E_t(P_{t^*+}|\hat{a}_{t^*} > \underline{a}(C), \ln(C)) \\
=& e^{-\phi(1-\bar{s})(\mu-(1+\phi)(1-\bar{s})(\sigma_r^2+\tau_\iota^2))(T-t^*)+\frac{\phi^2(1-\bar{s})^2\hat{\tau}^2_{a_{t^*}}}{2}(T-t^*)^2-\phi E(\Psi_{t^*})+\frac{\phi^2}{2}Var(\Psi_{t^*})} \\
& \times (1-F_N(\hat{a}_t-\phi Cov(\Psi_{t^*}, \hat{a}_{t^*}), \hat{\tau}_t-\hat{\tau}^2_{t^*}; \underline{a}(C))) \\
& \times e^{-\phi(1-\bar{s})(\mu-(1+\phi)(1-\bar{s})(\sigma_r^2+\tau_\iota^2))(T-t^*)+\frac{\phi^2(1-\bar{s})^2\tau_a^2}{2}(T-t^*)^2-\phi E_t(ln(S_{t^*}))+\frac{\phi^2}{2}Var(ln(S_{t^*})} \\
& \times F_N(\hat{a}_t-\phi Cov(ln(S_{t^*}), \hat{a}_{t^*}), \hat{\tau}^2_{a_t}-\hat{\tau}^2_{a_{t^*}}; \underline{a}(C)))
\end{aligned}$$

Because $\ln(C)$ only enters $\underline{a}(C)$, $E_t(P_{t^*+})$ is given by

$$\begin{aligned}
E_t(P_{t^*+}) =& q(\hat{a}_{t^*})E_t(P_{t^*+}|\hat{a}_{t^*} \leq \underline{a}(C), \ln(C)) + (1-q(\hat{a}_{t^*}))E_t(P_{t^*+}|\hat{a}_{t^*} > \underline{a}(C), \ln(C)) \\
=& e^{-\phi(1-\bar{s})(\mu-(1+\phi)(1-\bar{s})(\sigma_r^2+\tau_\iota^2))(T-t^*)+\frac{\phi^2(1-\bar{s})^2\hat{\tau}^2_{a_{t^*}}}{2}(T-t^*)^2-\phi E(\Psi_{t^*})+\frac{\phi^2}{2}Var(\Psi_{t^*})} \\
& \times (1-E_t(F_N(\hat{a}_t-\phi Cov(\Psi_{t^*}, \hat{a}_{t^*}), \hat{\tau}_t-\hat{\tau}^2_{t^*}; \underline{a}(C)))) \\
& \times e^{-\phi(1-\bar{s})(\mu-(1+\phi)(1-\bar{s})(\sigma_r^2+\tau_\iota^2))(T-t^*)+\frac{\phi^2(1-\bar{s})^2\tau_a^2}{2}(T-t^*)^2-\phi E_t(ln(S_{t^*}))+\frac{\phi^2}{2}Var(ln(S_{t^*})} \\
& \times E_t(F_N(\hat{a}_t-\phi Cov(ln(S_{t^*}), \hat{a}_{t^*}), \hat{\tau}^2_{a_t}-\hat{\tau}^2_{a_{t^*}}; \underline{a}(C)))
\end{aligned} \tag{IA.23}$$

Thus, we can obtain the expression of $Prob_t^{Change}$ and $Prob_t^{Stay}$ by calculating the expected probability terms in equation IA.23. ■

Derivation of $\sigma_{P,t}$ and K_P. When $t < t^*$, the state price density P_t is a martingale. Therefore, the volatility $\delta_{P,t}$ can be obtained by using Ito's Lemma on equation (27). When $t > t^*$, the expression of P_t depends on whether there is disclosure happening at t^* or not.

Based on derivation in the Proof of Proposition 4, the expression of $\delta_{P,t}$ can also be obtained by using Ito's Lemma.

K_P measures the size of the jump of the state price density at the time of t^*. Let $P^D_{t^*+}$ be the state price density when disclosure happens, and $P^{ND}_{t^*+}$ be the state price density when the firm choose not to disclose. Based on the derivation in the Proof of Proposition 4. The jump of the state price density can be calculated as

$$K^D_{P,t^*} = (\frac{P^D_{t^*+}}{P_{t^*}} - 1) = \frac{P^D_{t^*+}}{q(\hat{a}_{t^*})P^D_{t^*+} + (1-q(\hat{a}_{t^*}))P^{ND}_{t^*+}} - 1$$
$$= \frac{1}{q(\hat{a}_{t^*}) + (1-q(\hat{a}_{t^*}))e^{-\phi(1-\bar{s})\hat{a}_{t^*} + \frac{\phi^2(1-\bar{s})^2(T-t^*)^2(\hat{\tau}^2_{a_{t^*}} - \hat{\tau}^2_{a_t})}{2}}} - 1$$
$$= \frac{(1-H(\hat{a}_{t^*}))(1-q(\hat{a}_{t^*}))}{q(\hat{a}_{t^*}) + (1-q(\hat{a}_{t^*}))H(\hat{a}_{t^*})}$$

where $H(\hat{a}_{t^*}) = e^{-\phi(1-\bar{s})\hat{a}_{t^*} + \frac{\phi^2(1-\bar{s})^2(T-t^*)^2(\hat{\tau}^2_{a_{t^*}} - \hat{\tau}^2_{a_t})}{2}}$. Based on the martingale condition, the following equation is satisfied

$$E_{t^*}(K_{P,t^*}) = q(\hat{a}_{t^*})K^D_{P,t^*} + (1-q(\hat{a}_{t^*}))K^{ND}_{P,t^*} = 0 \tag{IA.24}$$

Therefore, we can derive the expression of K_P when there is no disclosure at t^*, which is K^{ND}_{P,t^*} in equation IA.24. ■

Proof of Proposition 5. This proof follows a similar structure of the proof of Proposition 4. When $t < t^*$, the market value of firm j's stock can be presented as

$$Q^j_t = E_t(\frac{P_{t^*+}Q^j_{t^*+}}{P_t})$$

Where P_t is known at time t, and $E_t(P_{t^*+}Q_{t^*+})$ when the disclosure change cost is C can be expressed as

$$
\begin{aligned}
E_t(P_{t^*+}Q^j_{t^*+}|\ln(C)) &= q_t(\ln(C))E_t(P_{t^*+}Q^j_{t^*+}|\hat{a}_{t^*} < \underline{a}(C), \ln(C)) \\
&\quad + (1-q_t(\ln(C)))E_t(P_{t^*+}Q^j_{t^*+}|\hat{a}_{t^*} > \underline{a}(C), \ln(C))
\end{aligned} \tag{IA.25}
$$

For the first conditional expectation for disclosure change, it can be transformed into

$$
\begin{aligned}
E_t(P_{t^*+}Q^j_{t^*+}|\hat{a}_{t^*} < \underline{a}(C), \ln(C)) &= E_t(S^{-\phi}_{t^*} e^{-\phi(1-\bar{s})(\mu-(1+\phi)(1-\bar{s})(\sigma_r^2+\tau_\iota^2))(T-t^*)+\frac{\phi^2(1-\bar{s})^2\tau_a^2(T-t^*)^2}{2}}) \\
&\quad \times S^j_{t^*} e^{(1-\bar{s})(\mu-\phi(1-\bar{s})(\sigma_r^2+\tau_\iota^2))(T-t^*)+\frac{1-2\phi}{2}(1-\bar{s})^2(T-t^*)^2\tau_a^2}|\hat{a}_{t^*} < \underline{a}(C), \ln(C)) \\
&= e^{(1-\phi)(1-\bar{s})(\mu-\phi(1-\bar{s})(\sigma_r^2+\tau_\iota^2))(T-t^*)+\frac{(1-\phi)^2}{2}((T-t^*)^2(1-\bar{s})\tau_a^2+(1-\bar{s})(\sigma_r+\tau_\iota^2)(T-t^*))} E_t(S^{-\phi}_{t^*}S^j_{t^*}|\hat{a}_{t^*} < \underline{a}(C), \ln(C))
\end{aligned} \tag{IA.26}
$$

According to the evolution of S_t and S^j_t before t^*, The conditional expectation in the last line of equation IA.26 can be presented as

$$
E_t(S^{-\phi}_{t^*}S^j_{t^*}|\hat{a}_{t^*} < \underline{a}(C), \ln(C)) = \frac{S^j_t}{S_t} E_t(S^{1-\phi}_{t^*}|\hat{a}_{t^*} < \underline{a}(C), \ln(C)) \tag{IA.27}
$$

Following a similar procedure we adopt in the proof of Proposition 4, equation IA.27 can be calculated as

$$
\begin{aligned}
\frac{S^j_t}{S_t} E_t(S^{1-\phi}_{t^*}|\hat{a}_{t^*} < \underline{a}(C), \ln(C)) &= \frac{F_N(\hat{a}_t + (1-\phi)(1-\bar{s})^2\hat{\tau}^2_{a_t}(t^*-t), \hat{\tau}^2_{a_t} - \hat{\tau}^2_{a_{t^*}}; \underline{a}(C))}{F_N(\hat{a}_t, \hat{\tau}^2_{a_t} - \hat{\tau}^2_{a_{t^*}}; \underline{a}(C))} \\
\times \frac{S^j_t}{S_t} S^{1-\phi}_t e^{(1-\phi)(1-\bar{s})\mu(T-t^*)+(1-\phi)(1-\bar{s})(t^*-t)\hat{a}_t - \frac{(1-\phi)\phi(\sigma_r^2+\tau_\iota^2)(T-t)}{2} + \frac{(1-\phi)^2(1-\bar{s})^2}{2}((T-t^*)^2\tau_a^2+(t^*-t)\hat{\tau}^2_{a_t})}
\end{aligned} \tag{IA.28}
$$

Plugging equation IA.28 into equation IA.26, $E_t(P_{t^*+}Q^j_{t^*+}|\hat{a}_{t^*} < \underline{a}(C), \ln(C))$ can be given as

$$
\begin{aligned}
E_t(P_{t^*+}Q^j_{t^*+}|\hat{a}_{t^*} < \underline{a}(C), \ln(C)) &= S^j_t S^{-\phi}_t e^{(1-\phi)(1-\bar{s})(\mu-\phi(1-\bar{s})(\sigma_r^2+\tau_\iota^2))(T-t^*)+\frac{(1-\phi)^2}{2}((T-t^*)^2(1-\bar{s})\tau_a^2+(1-\bar{s})(\sigma_r+\tau_\iota^2)(T-t^*))} \\
&\times e^{(1-\phi)(1-\bar{s})\mu(T-t^*)+(1-\phi)(1-\bar{s})(t^*-t)\hat{a}_t - \frac{(1-\phi)\phi(\sigma_r^2+\tau_\iota^2)(T-t)}{2} + \frac{(1-\phi)^2(1-\bar{s})^2}{2}((T-t^*)^2\tau_a^2+(t^*-t)\hat{\tau}^2_{a_t})} \\
&\times \frac{F_N(\hat{a}_t + (1-\phi)\hat{\tau}^2_{a_t}(t^*-t), \hat{\tau}^2_{a_t} - \hat{\tau}^2_{a_{t^*}}; \underline{a}(C))}{F_N(\hat{a}_t, \hat{\tau}^2_{a_t} - \hat{\tau}^2_{a_{t^*}}; \underline{a}(C))}
\end{aligned}
$$

The conditional expectation without disclosure change $E_t(P_{t^*+}Q^j_{t^*+}|\hat{a}_{t^*} \geq \underline{a}(C), \ln(C))$ can be presented in the following way using the expression of state price density and market value derived in the Proof of Proposition 4

$$\begin{aligned} E_t(P_{t^*+}Q^j_{t^*+}|\hat{a}_{t^*} \geq \underline{a}(C), \ln(C)) &= E_t(S_{t^*}^{-\phi} e^{-\phi(1-\bar{s})(\mu+\hat{a}_{t^*}-(1+\phi)(1-\bar{s})(\sigma_r^2+\tau_\iota^2))(T-t^*)+\frac{\phi^2(1-\bar{s})^2\tau_a^2(T-t^*)^2}{2}}) \\ &\times S^j_{t^*} e^{(1-\bar{s})(\mu+\hat{a}_{t^*}-\frac{(1-\bar{s})(\sigma_r^2+\tau_\iota^2)}{2})+\frac{(1-2\phi)(1-\bar{s})^2}{2}((T-t^*)(1-\bar{s})\hat{\tau}^2_{a_{t^*}}+(\sigma_r^2+\tau_\iota^2)(T-t^*))}|\hat{a}_{t^*} < \underline{a}(C), \ln(C)) \\ &= e^{(1-\phi)(1-\bar{s})\mu(T-t^*)-\frac{\phi(1-\phi)(1-\bar{s})^2(\sigma_r^2+\tau_\iota^2)(T-t^*)}{2}+\frac{(1-\phi)^2(1-\bar{s})^2\tau^2_{a_{t^*}}}{2}} E_t(S_{t^*}^{-\phi}S^j_{t^*}|\hat{a}_{t^*} < \underline{a}(C), \ln(C)) \end{aligned} \tag{IA.29}$$

By the same way we derive $E_t(P_{t^*+}Q^j_{t^*+}|\hat{a}_{t^*} < \underline{a}(C), \ln(C))$, equation IA.29 is equal to

$$\begin{aligned} E_t(P_{t^*+}Q^j_{t^*+}|\hat{a}_{t^*} \geq \underline{a}(C), \ln(C)) &= \frac{1 - F_N(\hat{a}_t + (1-\phi)Cov(\Psi_{t^*}, \hat{a}_{t^*}), \hat{\tau}^2_{a_t} - \hat{\tau}^2_{a_{t^*}}; \underline{a}(C))}{1 - F_N(\hat{a}_t, \hat{\tau}^2_{a_t} - \hat{\tau}^2_{a_{t^*}}; \underline{a}(C))} \\ \times S^j_t S_t^{-\phi} e^{(1-\phi)(1-\bar{s})\mu(T-t)-\frac{\phi(1-\phi)(1-\bar{s})^2(\sigma_r^2+\tau_\iota^2)(T-t)}{2}+(1-\phi)(1-\bar{s})\hat{a}_t(T-t)+\frac{(1-\phi)^2(1-\bar{s})^2\hat{\tau}^2_{a_t}(T-t)^2}{2}} \end{aligned}$$

Then an analytic expression for $E_t(P_{t^*+}Q^j_{t^*}|\ln(C))$ can be obtained by equation IA.25. And $E_t(P_{t^*+}Q^j_{t^*+})$ can be calculated by integrating $E_t(P_{t^*+}Q^j_{t^*}|\ln(C))$ over $\ln(C)$. The expression of $Prob^{Change}_{1t}$ and $Prob^{Stay}_{1t}$ can be obtained in a similar way to obtain $Prob^{Change}_t$ and $Prob^{Stay}_t$ in the proof of Proposition 4. ■

Derivation of $\mu_{Q,t}$, $\sigma_{Q,t}$ and K_Q. When $t < t^*$, the evolution of the increasing rate of market value $\frac{dQ^j_t}{Q^j_t}$ can be obtained by deriving the expression of the market value given by Proposition 5 using Ito's Lemma, which can give us the expression of $\mu_{Q,t}$ and $\sigma_{Q,t}$ before t^*.

When $t > t^*$, $\frac{dQ^j_t}{Q^j_t}$ can be obtained by the optimal choice problem of the shareholder. Using Ito's Lemma again, we can have the expression of $\mu_{Q,t}$ and $\sigma_{Q,t}$ after t^*.

For the jump K_Q at time t^*, we use the same strategy we used to derive the jump of state price density at t^*, K_J.

$$K^D_{Q,t^*} = \frac{Q^{j,New}_{t^*+} - Q^j_t}{Q^j_t} = \frac{(1-q(\hat{a}_{t^*}))H(\hat{a}_{t^*})(1-N_1(\hat{a}_{t^*}))}{q(a_{t^*}) + (1-q(\hat{a}_{t^*}))H(\hat{a}_{t^*}N_1(\hat{a}_{t^*}))}$$

where $Q^{j,New}_{t^*+}$ and Q^j_t are stock market value derived in the Proof of Proposition 4, K^D_{Q,t^*} is the jump at t^* when there is a disclosure change, and $N_1(\hat{a}_{t^*})$ is given in Section 2.3. By the same logic, K^{ND}_{Q,t^*} can be obtained by

$$K^{ND}_{Q,t^*} = \frac{Q^{j,Old}_{t^*+} - Q^j_t}{Q^j_t} = K^D_{Q,t^*} N_1(\hat{a}_{t^*}) + N_1(\hat{a}_{t^*}) - 1$$

Therefore, the drifts, diffusions and jumps of $\frac{dQ^j_t}{Q^j_t}$ illustrated in Section 2 are all obtained. ∎